GOVERNMENT POLYTECHNIC, SIRSA

BRANCH   : MECHANICAL ENGINEERING
SUBJECT   : THOERY OF MACHINES
SEMESTER : 5th

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CHAPTER-1
SIMPLE MECHANISMS

1.1 KINEMATICS OF MACHINES: - DEFINITION OF KINEMATICS, DYNAMICS, STATICS, KINETICS, KINEMATIC LINK, KINEMATIC PAIR AND ITS TYPES, CONSTRAINED MOTION AND ITS TYPES, KINEMATIC CHAIN AND ITS TYPES, MECHANISM, INVERSION, MACHINE AND STRUCTURE.

THEORY OF MACHINE

It deals with the study of relative motion between the various parts of machine and the forces which act on them. Theory of machine may be sub- divided into the following categories:

• **Kinematics of Machine:** It deals with the relative motion between the various parts neglecting the forces which are responsible for producing these motions.

• **Dynamics of Machine:** It deals with the forces act on various parts of machines. Dynamics of machine may be sub-divided into following two categories:

  ▪ **Static:** It deals with the study of forces which act on different parts of machines having the part must be in rest position.

  ▪ **Kinetics:** It deals with the inertia forces due to mass and motion of parts of machine.

KINEMATIC LINK

Each part of machine which moves relative to other is called kinematic link or element. If a link may consist of number of parts connected in such a way that they form one unit and have no relative motion with each other. Example piston, piston rod and cross head of engine constitute one unit and behave like single link. On the other hand cross head, connecting rod, frame of engine are different links.

Types of Links

- **Rigid link:** A rigid link is one which does not undergo any deformation while transmitting motion. In actual practice, no link is perfectly rigid, but for our convenience we may take connecting rod, crank etc. as rigid.

- **Flexible Link:** A flexible link is that which, while transmitting motion is partly deformed in such a manner that transmission is not affected.

- **Fluid Link:** It is formed by having fluid in a container and motion is transmitted through fluid by pressure. E.g. hydraulic Jack, hydraulic press etc.


KINEMATIC PAIR

A pair is formed when there is a contact between two links of machine, but when there is relative motion between the two links, then it is called kinematic pair. The relative motion between the two links must be completely or successfully constrained to make required pair. Examples are Cylinder and piston, piston and connecting rod etc.


Types of kinematic pairs:

(i) Based on nature of contact between elements:

(a) **Lower pair:** If the joint by which two members are connected has surface contact, the pair is known as lower pair. Eg. pin joints, shaft rotating in bush, slider in slider crank mechanism.
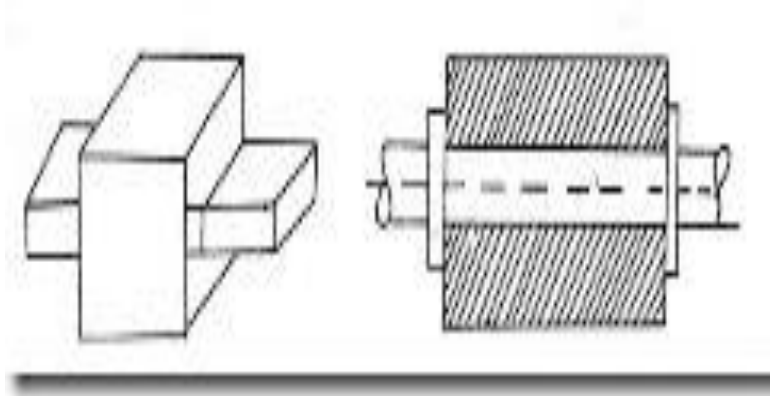
(b) **Higher pair:** If the contact between the pairing elements takes place at a point or along a line, such as in a ball bearing or between two gear teeth in contact, it is known as a higher pair.
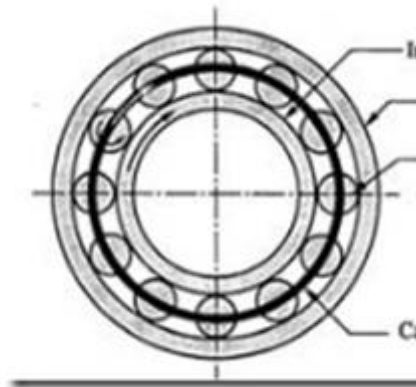
(ii) Based on relative motion between pairing elements:

   (a) **Siding pair:** Sliding pair is constituted by two elements so connected that one is constrained to have a sliding motion relative to the other.
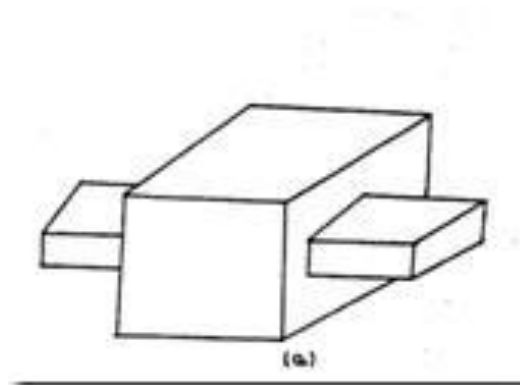
(b) **Turning pair**: When connections of the two elements are such that only a constrained motion of rotation of one element with respect to the other is possible, the pair constitutes a turning pair.
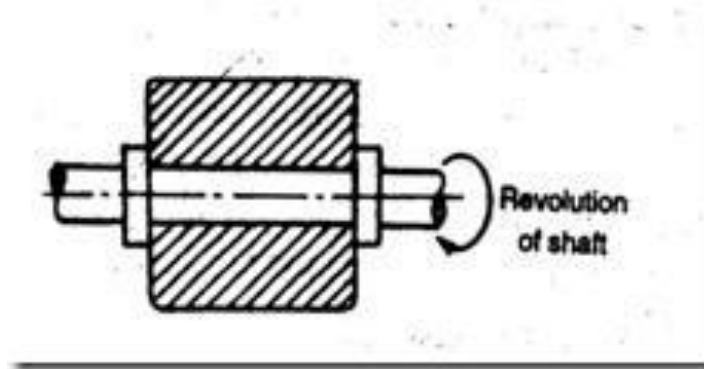
(c) Cylindrical pair: If the relative motion between the pairing elements is the combination of turning and sliding, then it is called as cylindrical pair.
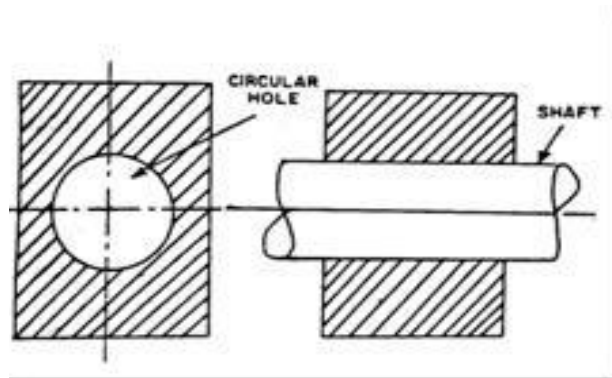
(d) Rolling pair: When the pairing elements have rolling contact, the pair formed is called rolling pair. E.g. Bearings, Belt and pulley.
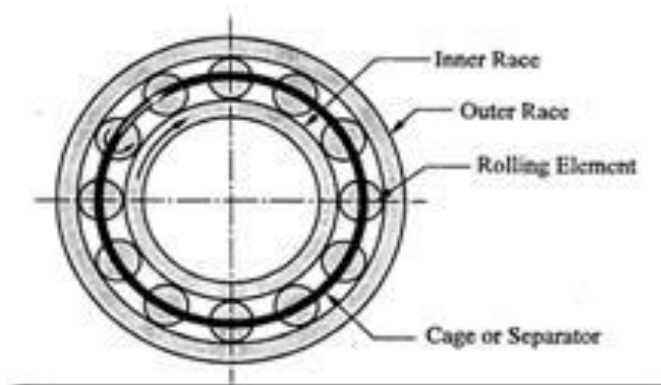
(e) **Spherical pair:** A spherical pair will have surface contact and three degrees of freedom. Eg. Ball and socket joint. DOF = 3

(f) **Helical pair or screw pair:** When the nature of contact between the elements of a pair is such that one element can turn about the other by screw threads, it is known as screw pair.

(b)
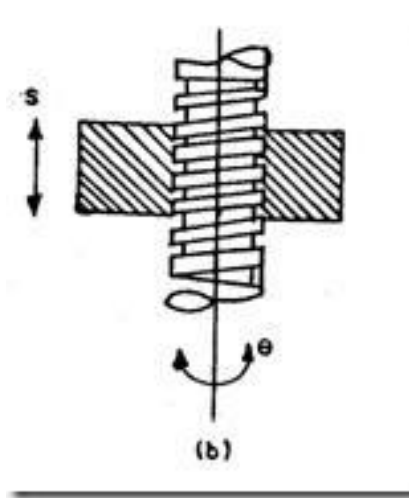
(iii) Based on the nature of mechanical constraint.

(a) **Closed pair:** Elements of pairs held together mechanically due to their geometry constitute a closed pair. They are also called form-closed or self-closed pair.

(a)
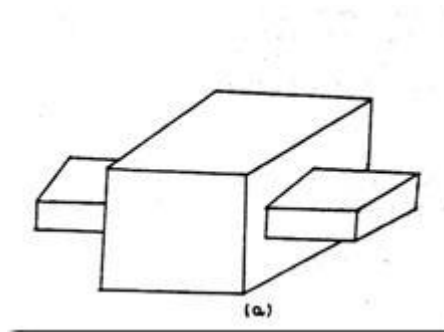
(b) **Unclosed or force closed pair**: Elements of pairs held together by the action of external forces constitute unclosed or force closed pair .Eg. Cam and follower.
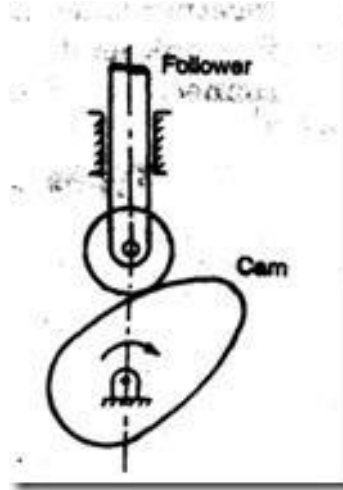
CONSTRAINED MOTION AND ITS TYPES

Constrained motion results when an object is forced to move in a restricted way i.e. moves in definite direction.

Constrained motion (or relative motion) can be broadly classified into three types:

1. Completely constrained motion
2. Incompletely constrained motion
3. Partially (or successfully) constrained motion

1. **Completely constrained motion:**

Completely constrained motion is a type of constrained motion in which relative motion between the links of a kinematic pair occurs in a definite direction by itself, irrespective of the external forces applied. E.g. Square bar in a square hole undergoes completely constrained motion
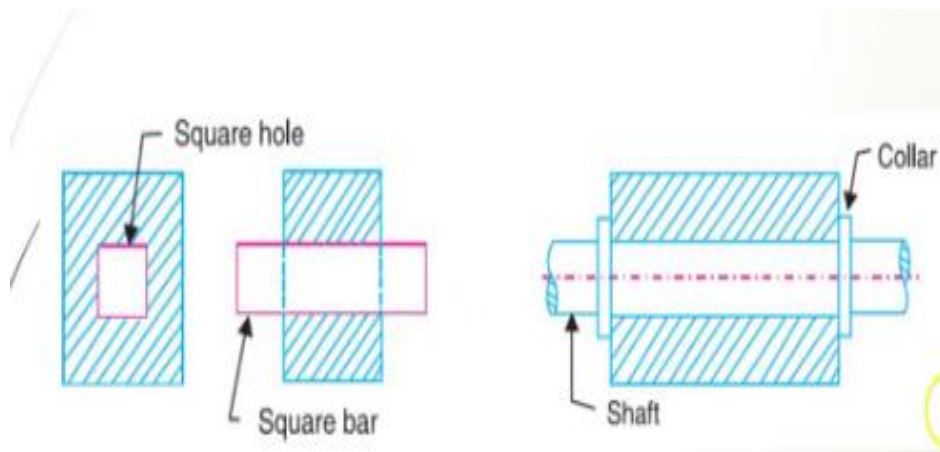
Even when any external force is applied, a square bar always slides inside a square hole. It does not turn.

2. Incompletely constrained motion:

In incompletely constrained motion, the relative motion between the links depends on the direction of external forces acting on them. A good example of incompletely constrained motion is the motion of a shaft inside a circular hole. Depending on the direction of external forces applied, the shaft may slide or turn (or do both) inside the circular hole. Incompletely constrained motion is undesirable in any mechanical system. It leads to improper mechanical outputs.
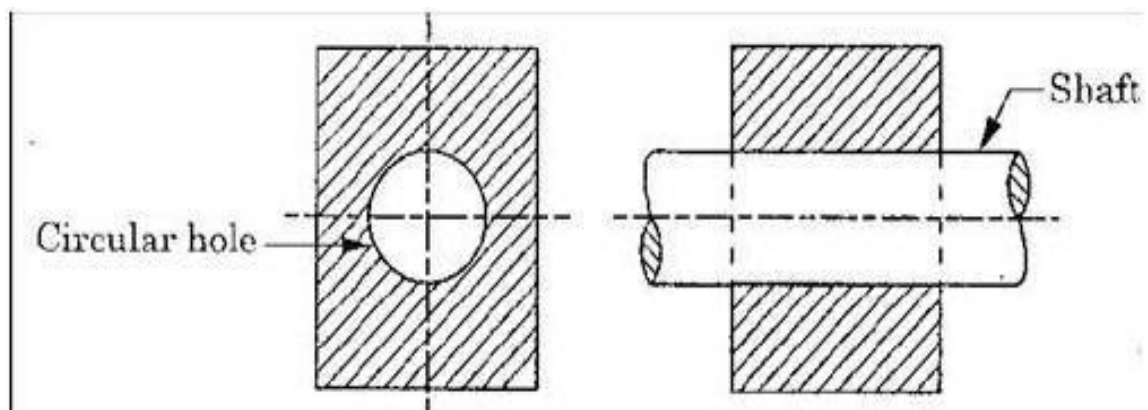
3. Partially (or successfully) constrained motion:

A kinematic pair is said to be partially or successfully constrained if the relative motion between its links occurs in a definite direction, not by itself, but by some other means. A good example of successfully constrained motion is piston reciprocating inside a cylinder in an internal combustion engine. Normally, when a piston is placed in a cylinder, it may undergo reciprocating motion (upward and downward motion) and turning motion, depending on the external forces applied. It is incompletely constrained.
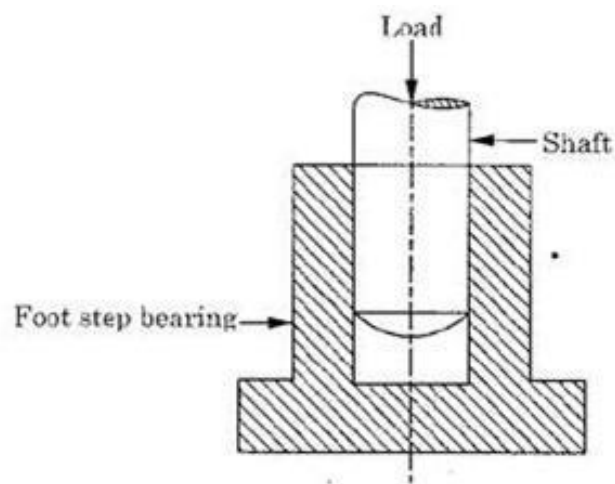
KINEMATIC CHAIN:

It is combination of kinematics links in such a way that the relative motion between the links is completely or successfully constrained and the last link is connected to the first link.

The following relationships hold good for a kinematic chain:

$$l = 2p - 4$$

$$j = \frac{3}{2}l - 2$$

Where,

l = Number of links

j = Number of joints

p = Number of pairs

Types of Kinematic Chain

1) Four Bar Chain
2) Slider Crank chain
3) Double Slider Crank Chain

1.) Four Bar Chain: It is made of four rigid links which are connected to form a quadrilateral by four pin-joints.
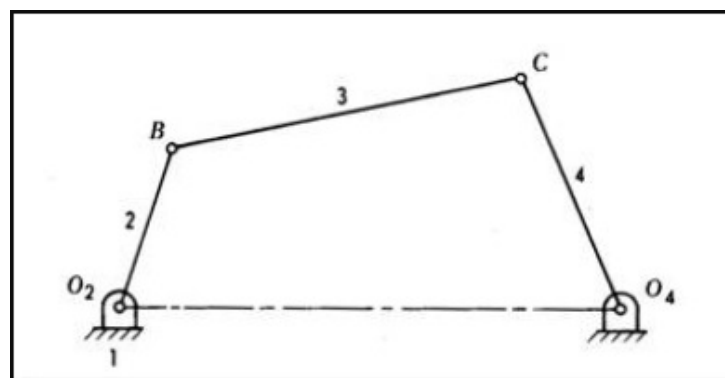
Link O_2O_4 is fixed and known as frame, link O_2B rotate and is called crank. The link BC is called coupler and link CO_4 is called lever.

Applications of Four Bar Chain:

1) Beam Engine
2) Coupled Wheel of locomotive
3) Engine indicator

2.) Slider Crank Chain: It is a four bar chain having one sliding pair and three turning pairs. This mechanism is used to convert reciprocating motion of piston into rotary motion of crank.
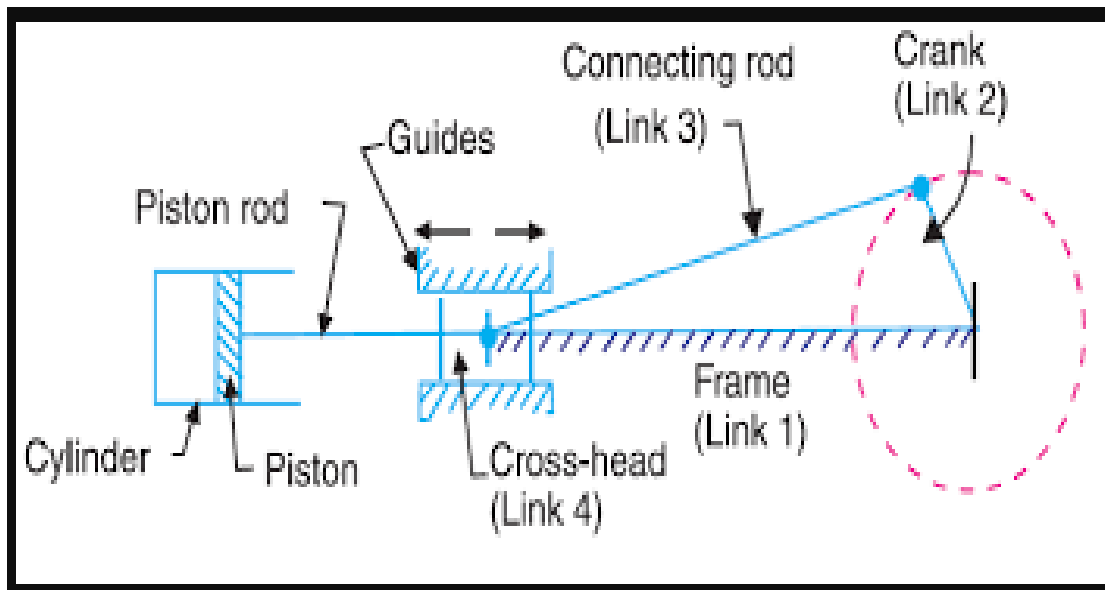
3.) Double Slider Crank Chain: A four bar chain with two turning pairs and two sliding pairs is known as double slider crank chain.
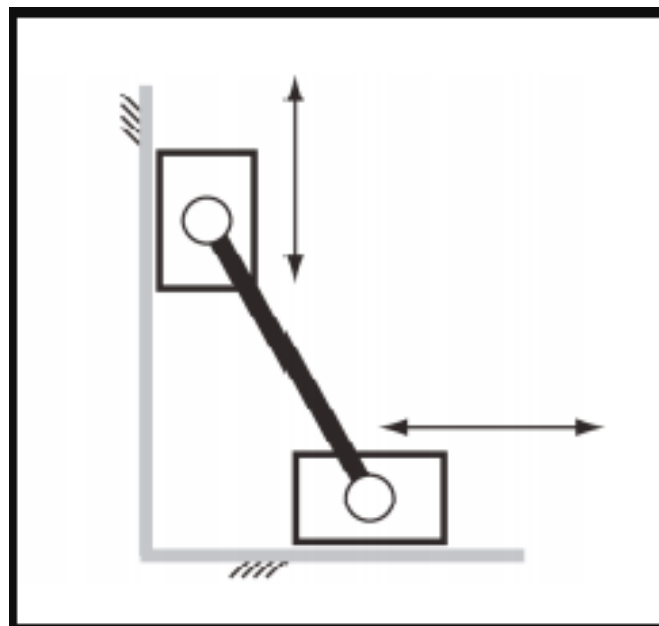
MECHANISM

It is an assembly of a number of resistant links which are connected in such a way that they have completely or successfully constrained relative motion between them and mechanism is obtained by fixing one of the links of kinematic chain.

A mechanism which has up to four links with lower pairs is called Simple mechanism.

A mechanism with more than four links is termed as compound mechanism.

MACHINE AND STRUCTURE

Machine: A machine is a device which receives energy in some available form and uses it to do some work. There exists a relative motion between its parts. E.g. lathe, shaper etc.

Structure: It is an assembly of various parts which having no relative motion between them and use to take up loads having straining actions. There exists no relative motion between its parts. E.g. roof, trusses, buildings etc.

INVERSION:

As a mechanism is obtained by fixing one of the link of kinematic chain. If any other link of the mechanism is fixed instead of the original one, the new mechanism obtained is called inversion of mechanism. Hence, different inversions can be obtained by fixing different links rather than original one. e.g. a slider crank chain become an oscillating cylinder mechanism if the connecting rod is fixed and a pendulum pump if the slider block is fixed.

1.2 INVERSIONS OF KINEMATIC CHAIN: INVERSION OF FOUR BAR CHAIN, COUPLED WHEELS OF LOCOMOTIVE & PANTOGRAPH. INVERSION OF SINGLE SLIDER CRANK CHAIN- ROTARY I.C. ENGINES MECHANISM, CRANK AND SLOTTED LEVER QUICK RETURN MECHANISM. INVERSION OF DOUBLE SLIDER CRANK CHAIN- SCOTCH YOKE MECHANISM & OLDHAM'S COUPLING.

INVERSION OF FOUR BAR CHAIN

1) **Coupled wheels of locomotive:** This mechanism is used to transmit rotary motion of one wheel to another wheel. In this link 1 is fixed and links 2 & 4 are of equal lengths and act as Crank. Link 3 act as connecting rod.
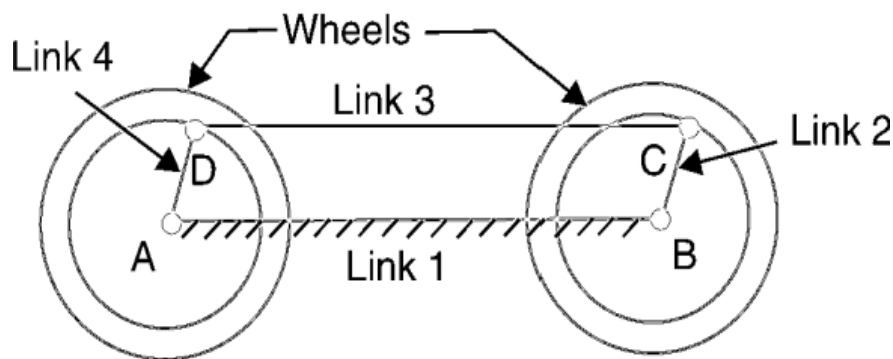
2) **Pantograph:** It is a mechanical linkage connected in a manner based on parallelograms so that the movement of one pen, in tracing an image, produces identical movements in a second pen. If a line drawing is traced by the first point, an identical, enlarged, or miniaturized copy will be drawn by a pen fixed to the other. Using the same principle, different kinds of pantographs are used for other forms of duplication in areas such as sculpture, minting, engraving, and milling.
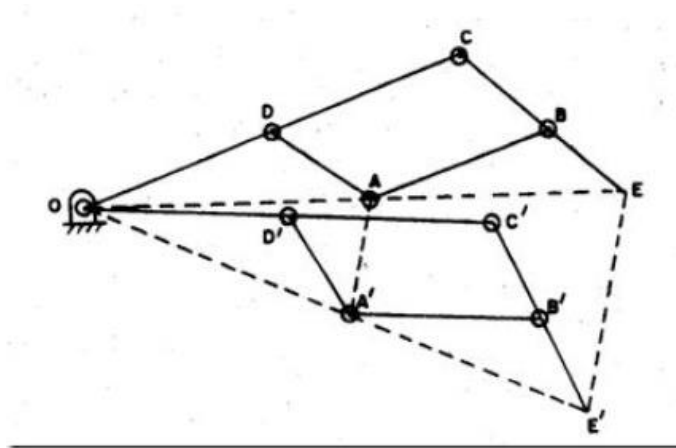
3) Beam Engine: It consists of four links. When the link 2 i.e. crank is rotated about point A then link 4 i.e. lever oscillates about point D. This mechanism is used to convert rotary motion into reciprocating motion.
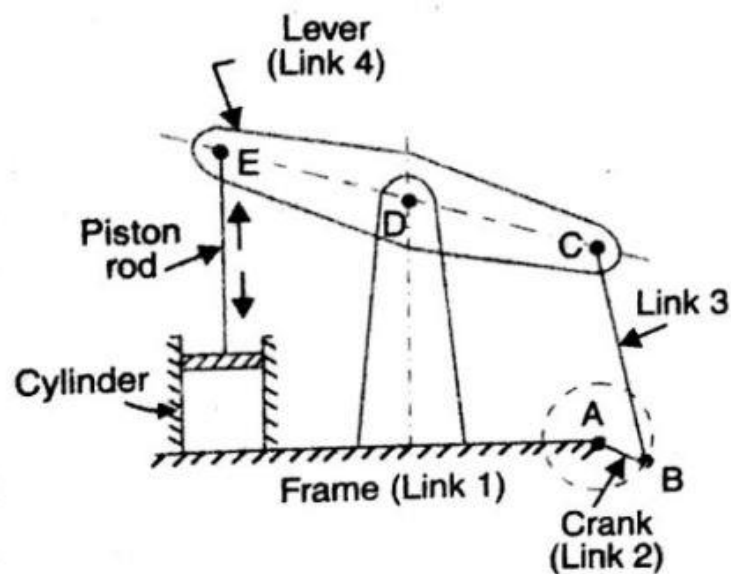
INVERSION OF SINGLE SLIDER CRANK CHAIN

1) Rotary I.C. Engines Mechanism

A single slider crank chain is a modification of the basic four-bar chain. It consist of one sliding pair and three turning pairs. It is, usually, found in reciprocating steam engine mechanism. This type of mechanism converts rotary motion into reciprocating motion and vice versa. In a single slider

crank chain, as shown the links 1 and 2, links 2 and 3, and links 3 and 4 form three turning pairs while the links 4 and 1 form a sliding pair.

Link 1 corresponds to the frame of the engine, which is fixed. The link 2 corresponds to the crank; link 3 corresponds to the connecting rod and link 4 corresponds to cross-head. As the crank rotates, the cross-head reciprocates in the guides and thus the piston reciprocates in the cylinder.
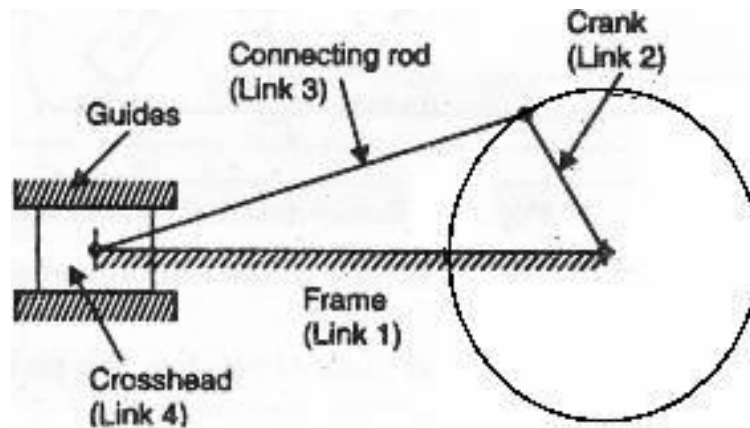
2) Bull engine or pendulum engine:

In this mechanism, the inversion is obtained by fixing the cylinder or link 4 (i.e. sliding pair), as shown in Fig. In this case, when the crank (link 2) rotates, the connecting rod (link 3) oscillates about a pin pivoted to the fixed link 4 at A and the piston attached to the piston rod (link 1) reciprocates. The duplex pump which is used to supply feed water to boilers have two pistons attached to link 1, as shown in Fig.
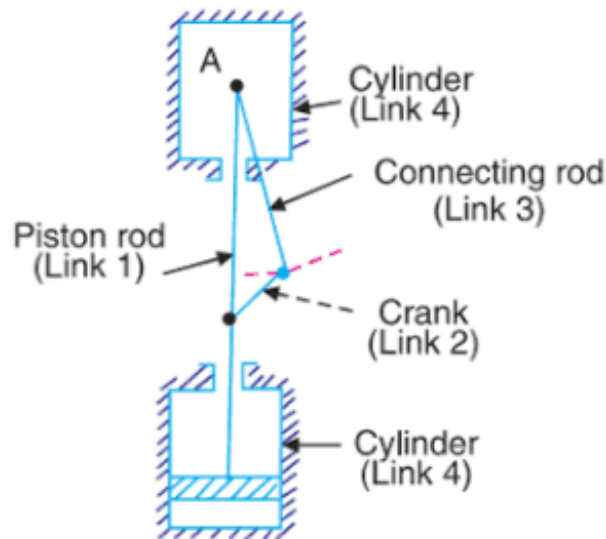
Pendulum Engine

3) Oscillating Cylinder Engine

The arrangement of oscillating cylinder engine mechanism, as shown in Fig. is used to convert reciprocating motion into rotary motion. In this mechanism, the link 3 forming the turning pair is fixed. Link 3 corresponds to the connecting rod of a reciprocating steam engine mechanism. When the crank (link 2) rotates, the piston attached to the piston rod (link 1) reciprocates and the cylinder (link 4) oscillates about a pin pivoted to the fixed link at A.
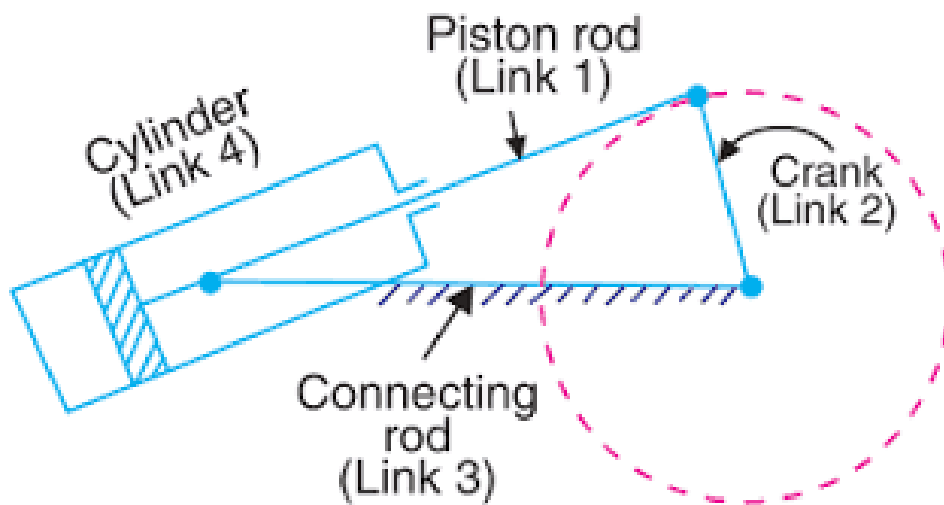
Oscillatory engine

INVERSION OF DOUBLE SLIDER CRANK CHAIN

1) Scotch Yoke Mechanism

In this mechanism, one of the sliding block is fixed. Rotary motion is given to crank about the fixed block.

Frame (Link 4)
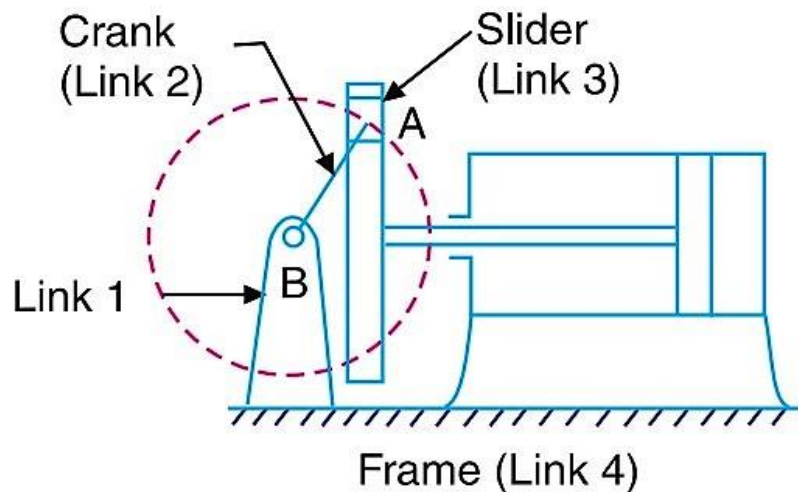
2) Oldham's Coupling

It means for transmissions of power from one shaft to another shaft parallel to each other when the distance between the two axes is small and variable. The shafts are connected in such ways that if one rotate, other shaft rotates at same speed and in the same direction.
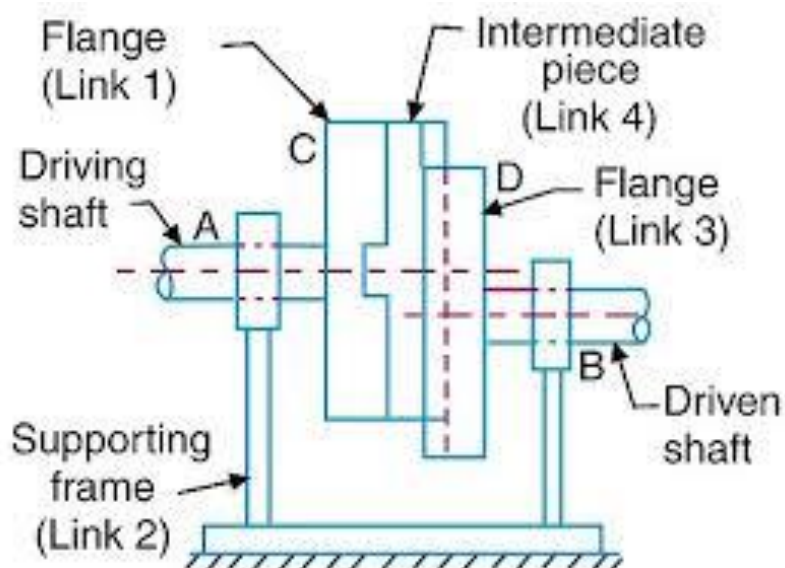
3) Elliptical Trammel

It is a device used for drawing ellipse. It has two grooves cut at right angle in a fixed plate. In the grooves, two sliding blocks are fitted. The link 2 joins the sliding blocks, or on its extension traces out an ellipse on the fixed plate.
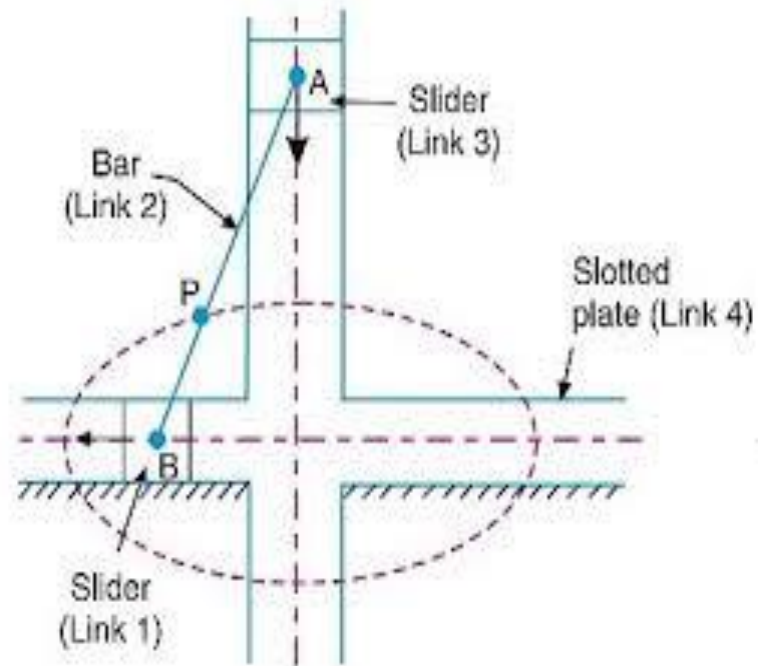
CHAPTER-2
POWER TRANSMISSION

2.1 INTRODUCTION TO BELT AND ROPE DRIVE

Power is transmitted from one shaft to another shaft by means of ropes, belts, chain and gears. For long distance belts, ropes or chain is used while for small distance gears are used.

Belt: The Flexible wrapping connectors are used to transmit power from one shaft to another shaft. When the wrapping connector takes the form of band, whose thickness is small in comparison to width is called belt.

Rope: The Flexible wrapping connectors are used to transmit power from one shaft to another shaft. If the cross section of wrapping connector is circular then it is called Rope.

The belt and rope must be kept in tension so that the motion is transmitted without any slip. Due to slipping and straining action belt and ropes are not positive types of drives as their velocity ratio is not constant.
In case of belt drive either flat or grooved pulley are used while in case of rope drive grooved pulley is used.

TYPES OF BELTS
- **Flat Belt:** A belt having rectangular cross section is known as flat belt. It is mostly used for power transmission at a distance not more than 8 meters.

- **V- Belt:** A belt having trapezoidal cross section is known as V - belt. It is mostly used for power transmission when two pulleys are close to each other. To take advantage of wedge cross section grooved pulley is used. Due to its wedging action it can transmit more power than flat pulley.\

- **Circular Belt:** It is also known as Rope. A belt having circular cross section is known as flat belt. It is mostly used for power transmission at a distance more than 8 meters

MATERIALS USED FOR BELTS

The materials used for belts must be strong, flexible and durable. It should have a high coefficient of friction. The various materials used are:

- **Leather:** The leather may be oak-tanned or mineral salt tanned ex: Chrome tanned. When the thickness of the belt required is more than, two or more strips are cemented together. Leather belts require periodic cleaning.

- **Fabric:** Fabric belts are made by folding canvas or cotton ducks is a layer (depending on the required thickness) and stitching together.

- **Rubber:** The belts are made of Fabric with a rubber layer. These are used in sawmills, paper mills, etc.

- **Balata:** The belts are made out of these materials are similar to rubber belts expect that balata gym is used instead of rubber. The belts of these materials are acid and waterproof but cannot be used where the temperature is above 45°.

2.2 TYPES OF BELT DRIVE

The following are the five main types of Belt Drives:

- Open belt drive.
- Cross belt drive.
- Stepped cone pulley or speed cone drive.
- Fast and loose pulleys.
- Jockey pulley drive.

Open Belt drive: In these types of belt drive, the belt is employing when the two parallel shafts have to rotate in the same direction. When the shafts are far apart, the lower side of the belt should be the tight side and the upper side must be the slack side. . It is mostly used for power transmission at a distance not more than 8 meters
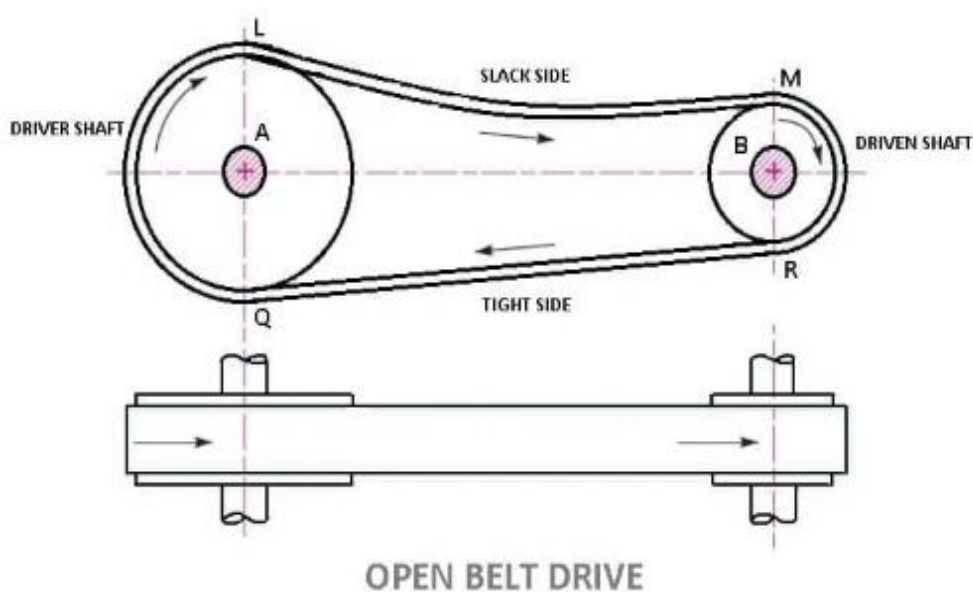
OPEN BELT DRIVE

Crossed belt drive: This type of belt drives, the belt is employing when two parallel shafts have to rotate in the opposite direction. At the junction where the belts cross, it rubs against itself and wears off. To avoid excessive wear, the shafts must be placed at a maximum distance from each other and operated at very low speeds.

CROSS BELT DRIVE
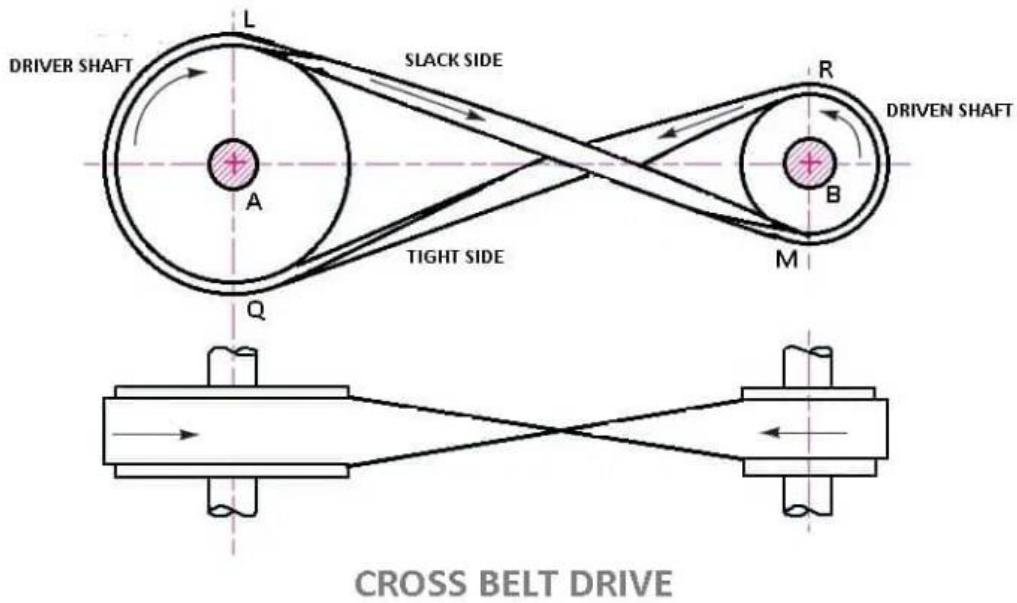
Steeped Cone Pulley or speed cone drive: This type of belt drives are used when the speed of the driven shaft is to be changed very frequently as in the case of machine tools such as lathe, drilling machine, etc.

STEPPED OR SPEED CONE PULLEY
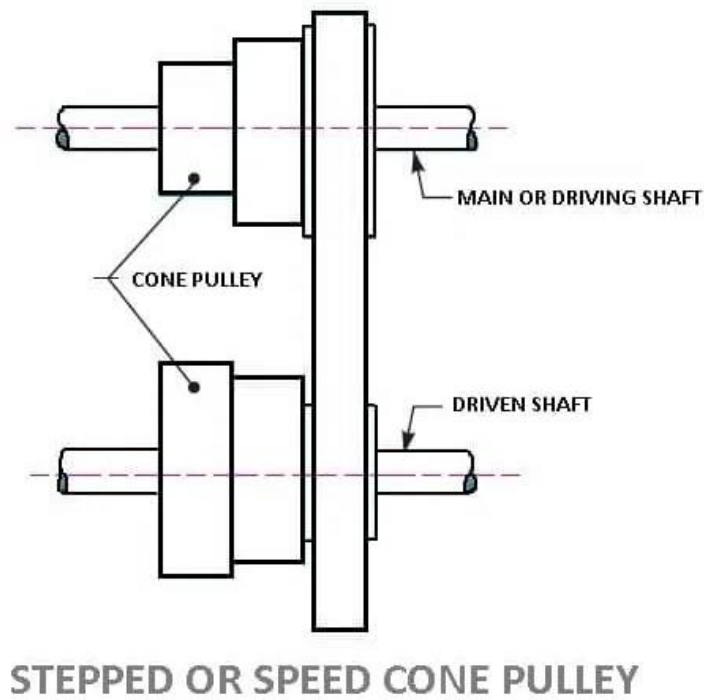
A stepped cone pulley is an integral casting having three or number of pulleys of different sizes one adjacent to the other. One set of stepped cone pulley is mounted in reverse on the driven shaft. An endless belt will be wrapped around one pair of pulleys. By shifting the belt from one pair of pulleys to the other, the speed of the driven shaft can be varied. The diameter of the driving and driven pulleys is such that the same belt will operate when shifted on different pairs of pulleys.

Fast and Loose Pulley: This types of belt drives are used when the driven or machine shaft is to be started or stopped whenever desired without interfering with the driving shaft. A pulley which is keyed to the machine shaft is called a fast pulley and run at the same speed as that of the machine shaft. A loose pulley runs freely over the machine shaft and is incapable of transmitting any power. When the driven shaft is required to be stopped, the belt is pushed on to the loose pulley by means of a sliding bar having belt forks.

FAST AND LOOSE PULLEY DRIVE
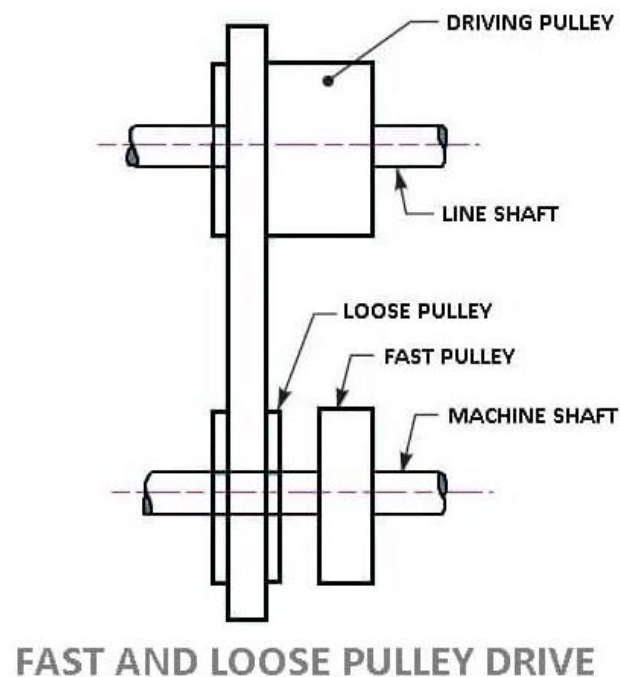
Jockey Pulley drive: In an open belt drive arrangement, if the center distance is small, or if the driven pulleys are very small, then the arc of contact of the belt with the driven pulley will be very small, which reduces the tensions in the belt, or if the required tension of the belt cannot be obtained by other means, an idler pulley, called jockey pulley is placed on the slack side of the belt. With increases the arc of contact and thus the tension which results in increased power transmission.
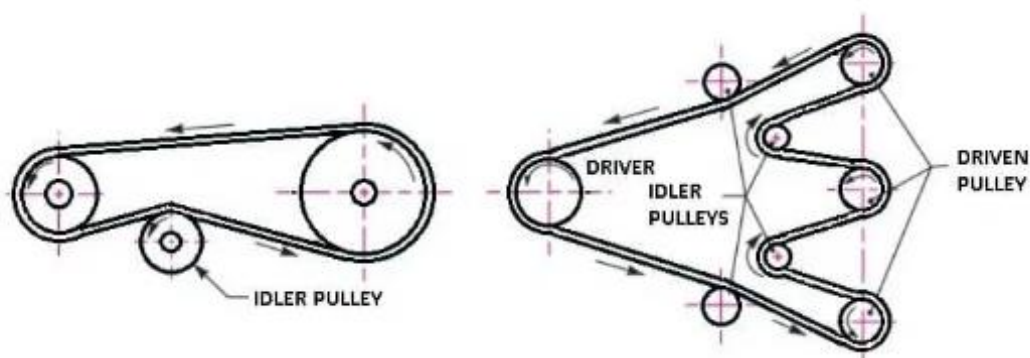
JOCKEY PULLEY DRIVE

2.3 CONCEPT OF VELOCITY RATIO, SLIP AND CREEP; CROWNING OF PULLEYS (SIMPLE NUMERICALS)

VELOCITY RATIO

It is the ratio of speed of the driven pulley to that of driving pulley.

Linear Speed of driver pulley $= \pi D_1 N_1$

Linear Speed of driven pulley $= \pi D_2 N_2$

Neglecting thickness of belt and also there is no slip between belt and pulley, then

$$\textbf{Velocity Ratio} = \frac{N2}{N1} = \frac{D1}{D2}$$

If we considering the thickness of the belt (t)

$$\textbf{Velocity Ratio} = \frac{N2}{N1} = \frac{D1+t}{D2+t}$$

SLIP

If the difference in tension between tight and slack sides if belt is too large to be resisted by friction between the belt and the pulley, then whole portion of belt which is in contact with the pulley begins to slide. When this frictional grip becomes insufficient, there is a possibility of forwarding motion of driver without pulley with it, this is known as the slip in a belt. Therefore slip may be defined as the relative motion between the pulley and the belt in it. This reduces the velocity ratio and usually expressed in % and it is denoted by S.

Slip at driver, S_1, Expressed in % of driver speed, $S_1\% = \dfrac{V_1 - V_b}{V_1} \times 100$

Slack side

Tight side

Slip at driven, S_2, Expressed in % of belt speed

$$S_2\% = \dfrac{V_b - V_2}{V_b} \times 100$$
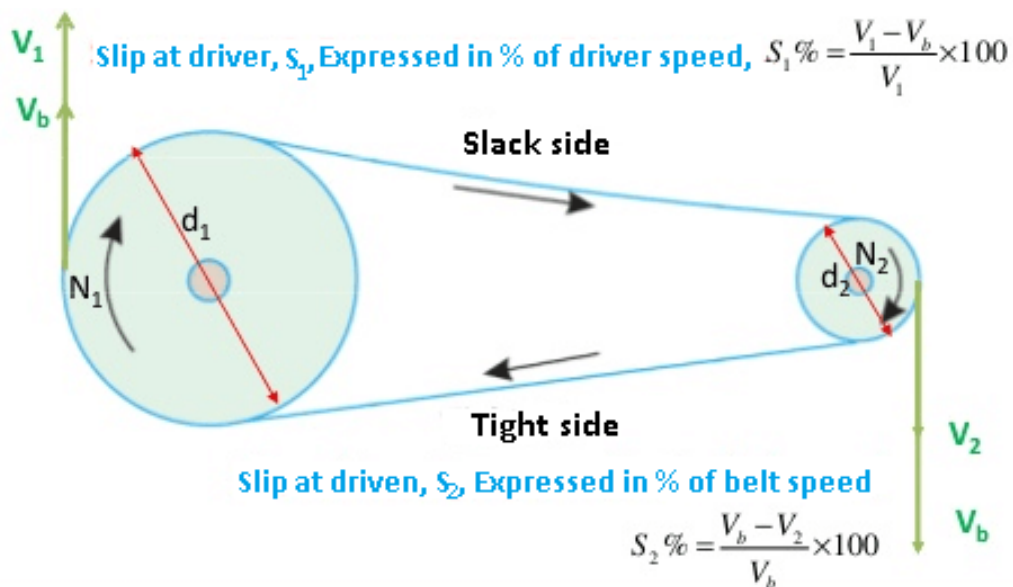
CREEP IN BELT

During rotation, there is an expansion of a belt on a tight side and contraction of the belt on the slack side. Due to this uneven expansion and contraction of the belt over the pulleys, there will be a relative movement (motion) of the belt over the pulleys this phenomenon is known as Creep in a belt.

Note: D1 and D2 are the Pitch Diameters
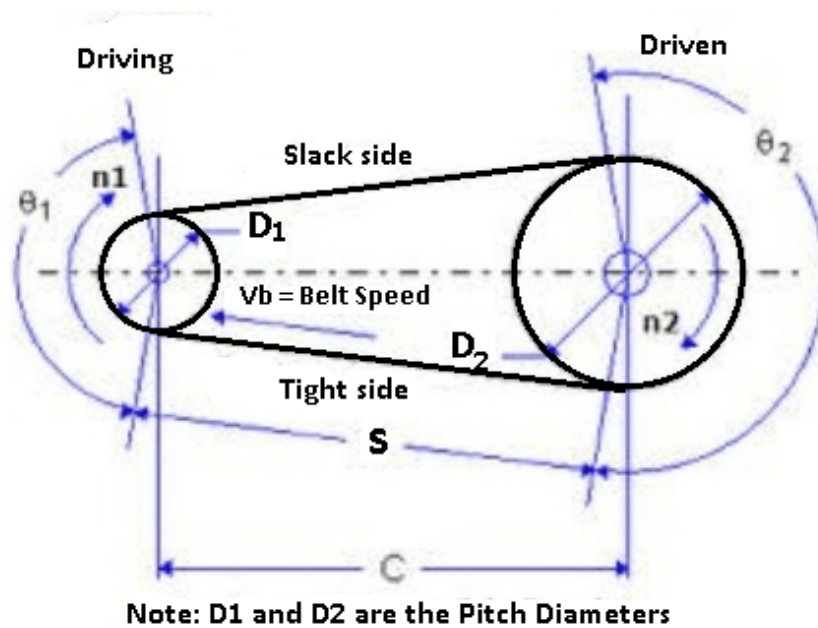
2.4 FLAT AND V BELT DRIVE: RATIO OF DRIVING TENSIONS, POWER TRANSMITTED, CENTRIFUGAL TENSION AND CONDITION FOR MAXIMUM HORSE POWER (SIMPLE NUMERICALS)

RATIO OF DRIVING TENSION

Ratio of Driving Tension for Flat Belt: Let us consider a driven pulley rotating in clockwise direction as shown in figure:
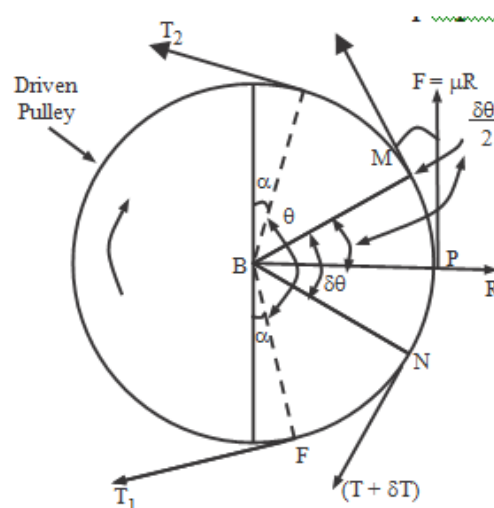
Let,

T_1 = Tension in belt on the tight side

T_2 = Tension in belt on the slack side

θ = Angle of contact

μ = Co efficient of friction existing between belt and pulley.

α = Angle of Lap

Consider driven or follower pulley. Belt remains in contact with the EBF.

Let T_1 and T_2 are the tensions in tight side and slack side.

Angle EBF known as angle of contact = $\prod$.-2α

Consider driven or follower pulley.

Belt remains in contact with the NPM. Let T_1 and T_2 are tensions in tight side and slack side. Let T be tension at point M & (T + dT) be the tension at point N.

Consider equilibrium in horizontal Reaction 'R' and vertical reaction μR.

Since the whole system is in equilibrium, that is

ΣV = 0; Tsin (90 - δq/2) + μR - (T + δT)sin(90 - δθ/2) = 0

Tcos (δθ/2) + μR = (T + δT) cos (δθ/2)

Tcos (δθ/2) + μR = Tcos(δq/2) + δTcos(δθ/2)

μR = δTcos(δθ/2)

Since δθ/2 is very small and cos0° = 1, So cos(δθ/2) = 1

μR= δT   ...(*i*)

ΣH = 0;

R-Tcos(90 - δq/2)-(T + δT)cos(90 - δθ/2) = 0

R = Tsin(δθ/2) + (T + δT)sin(δθ/2) As δθ/2 is small So sin(δθ/2) = δθ/2

R = T(δθ/2) + T(δθ/2) + δT(δθ/2) R = T.δθ + δT(δθ/2)

As δT(δq/2) is very small So δT(δq/2) = 0

R= T.δq   ...(*ii*)

Putting value of (*ii*) in equation (*i*)

μ.T.δθ = δT

or, $\delta T/T = \mu.\delta\theta$

Integrating both theside: , Where θ = Total angle of contact

$$\int_{T_2}^{T_1} \delta T/T = \mu \int_{0}^{0} \delta\theta,$$

$\ln(T_1/T_2) = \mu$

$$\frac{T1}{T2} = e^{\mu.\theta}$$

Thus, Ratio of belt tension $= \dfrac{T1}{T2} = e^{\mu.\theta}$

Ratio of Driving Tension for V- Belt

Let

RN = Normal reaction between belt and sides with a grooved pulley.

2α = Angle of groove

μ = Co-efficient of friction between belt and pulley

R = Total reaction in the plane of groove.

Resolving the forces vertically,

We get $R = R_N \sin\alpha + R_N \sin\alpha = 2R_N \sin\alpha$

$R_N = (R/2) \operatorname{cosec}\alpha$ .....(i)

Frictional resistance $= \mu R_N + \mu R_N = 2\mu R_N = 2\mu(R/2)\operatorname{cosec}\alpha$

$= \mu R \operatorname{cosec}\alpha = R\,\mu\operatorname{cosec}\alpha$

So, Ratio of Tension in V-Belt: $\dfrac{T1}{T2} = e^{\mu.\theta.\operatorname{cosec}\alpha}$

POWER TRANSMITTED BY A BELT

Let,

T_1 = Tension on tight side of Belt

T_2 = Tension on slack side of Belt

v = Linear velocity of belt

P = Power transmitted by Belt

$$P = (T_1 - T_2)v \quad \text{Watt}$$

CENTRIFUGAL TENSION

When the belt continuously moves over the pulley, the centrifugal force due to its own weight tends to fit in from the pulley. This centrifugal force produce equal tensions on tight side and slack side of the belt is known as Centrifugal tension.

Consider a short element of belt.

Let

m = mass per unit length of belt

T_c = centrifugal tension on tight and slack sides of element

F_c = centrifugal force on the element

r = radius of the pulley

v = velocity of the belt

$\delta\theta$ = angle of lap of the element over the pulley

F_c = mass of element x acceleration

= (length of element x mass per unit length) x acceleration

$= (r\delta\theta \times m) \times \dfrac{v2}{r}$

$= mv^2 \delta\theta \qquad\qquad$ (i)

Also,

$$F_c = 2\, T_c\, \sin\dfrac{\delta\theta}{2}$$

As $\delta\theta$ is small,

$$\text{Sin}\,\dfrac{\delta\theta}{2} \approx \dfrac{\delta\theta}{2}$$

$$F_c = 2\, T_c \dfrac{\delta\theta}{2}$$

$$F_c = T_c\, \delta\theta \qquad\qquad \text{(ii)}$$

From (i) and (ii)

$$Tc \, \delta\theta = mv^2\delta\theta$$

Or **$Tc = mv^2$**

Thus centrifugal tension is independent of the tight and slack side tensions and depends only on the velocity of the belt over the pulley.

CONDITION FOR MAXIMUM POWER TRANSMISSION

If it is desired that a belt transmits maximum possible power, two conditions must be fulfilled simultaneously. They are:

1. Larger tension must reach the maximum permissible value for the belt
2. The belt should be on the point of slipping, i.e. maximum frictional force is developed in the belt.

Let,

$T1$ = Tension on tight side of Belt

T_2 = Tension on slack side of Belt

v = Linear velocity of belt

P = Power transmitted by Belt

$$P = (T_1 - T_2) \cdot V \qquad \qquad \dots(i)$$

But we know that $T_1/T_2 = e^{\mu\theta}$

Or we can say that $T_2 = T_1/e^{\mu\theta}$

Putting the value of T_2 in equation (i)

$$P = (T_1 \cdot T_1/e^{\mu\theta}) \cdot v = T_1(1 - 1/e^{\mu\theta}) \cdot V \qquad \qquad \dots(ii)$$

Let $(1 - 1/e^{\mu\theta}) = K$, K = any constant

Then the above equation is $P = T_1 \cdot K \cdot V$ or $KT_1 V$ $\qquad \qquad \dots(iii)$

Let T_{max} = Maximum tension in the belt

T_c = Centrifugal tension which is equal to $m \cdot v^2$

Then $T_{max} = T_1 + T_c$

$$T_1 = T_{max} - T_c$$

Putting this value in the equation (iii)

$$P = K(T_{max} - T_c) \cdot V$$
$$= K(T_{max} - m \cdot V^2) \cdot V$$
$$= K(T_{max} \cdot v - m \cdot V^3)$$

Power transmitted will be maximum if $d(P)/dv = 0$

Thus differentiating equation with respect to V and equating to zero for maximum power, we get

$$d(P)/dv = K(T_{max} - 3 \cdot m \cdot V^2) = 0$$
$$T_{max} - 3mV^2 = 0$$
$$T_{max} = 3mV^2$$
$$V = (T_{max}/3m)^{1/2}$$

Equation (iv) gives velocity of belt at which maximum power is transmitted.

From equation (iv)

$T_{max} = 3T_c$ ………………………………………………………………..(v)

Hence when power transmitted is maximum, the centrifugal tension would be around $1/3^{rd}$ of the maximum tension.

We know that $T_{max} = T_1 + T_c$

$$= T_1 + T_{max}/3$$
$$T_1 = T_{max} - T_{max}/3$$
$$= 2/3.T_{max}$$

Hence condition for transmission of maximum power is:

$T_c = 1/3\ T_{max}$, and

$T_1 = 2/3T_{max}$

2.5 DIFFERENT TYPES OF CHAINS AND THEIR TERMINOLOGY

CHAIN DRIVE

A chain drive consist of an endless chain running over two sprocket, driver and driven. The velocity ratio is constant. The chains are mostly used to transmit motion and power from one shaft to another, when the centre distance between the shafts is short such as in bicycle, motorcycle etc.

DIFFERENT TYPES OF CHAINS

There are different types of chains used in power transmission.

- Hoisting Chains
- Conveyor Chains
- Power transmission Chains.

Hoisting Chains: A hoist is a mechanical device which is used to lift a load or lowering a load, it can be used for shifting of some heavy product from one place to another place in a work station. Hoist chains can be classified into two categories:

    a) Oval-Link Chains

    b) Stud-Link Chains

  a) **Oval-Link Chains**: It is one of the common types of chain used in hoist; it consists of many oval links attached to each other. These types of

chains are also called Coil Chains. Links of this type of chain is oval. However, there are square link types of chains that are also available but the kinking is occurred easily due to high loading. Generally, we used these types of chains at low speeds only.

b) **Stud-Link Chains:** The stud-link chains used to minimize the deformation and link or tangle easily. In this type of chain, a round bar or stud is used to fit inside the oval-link chains to provide more strength to the chain. It is used in Ship to up and down the anchor of the ships, and some crane hoist where we need to lift a very high amount of load.

Conveyor Chains: As the name suggests conveyor chains, which means these types of chains are mostly used in the conveyor. If you don't know the conveyor, it is a mechanical device system that is used to move the materials from one place to another. This type of chain is well shaped that it can easily run over the sprocket.

Conveyor Chains are categorized into three types:

  a) Detachable or Hook Joint types Conveyor chain.
  b) Closed-end pintle type conveyor chain.

a) **Detachable or Hook Joint types Conveyor chain**: Detachable or Hook Joint types Conveyor chains are used in a conveyor where the length between power transmissions is short. Detachable or Hook Joint types Conveyor chains are used in a conveyor where the length between power transmissions is short.

b) **Closed-end pintle type conveyor chain**: This type of chain is consists of a barrel and link and made in a single casting, and then the chain is heat-treated to provide higher strength.

Power Transmission Chains: You can easily find out by name of the chain that it is used to transmit the power. This type of chain is made of steel and sometimes it heats treated to minimize wear and tear. This type of chain has greater accuracy and can easily run over the sprocket. Power Transmission Chains are categorized into three types:

     a) Block Chain

     b) Roller Chain

     c) Silent Chain or Inverted tooth Chain

a) **Block Chain**: This type of chain is used in low-speed areas and will produce noise due to the sudden contact between sprocket and chain, however, this is used in some low-speed conveyor machines.

b) **Roller Chain:** This type of chain is constructed by:

- A bush
- Inner Link
- A pin
- Outer Plate
- Inner plate

In this type of chain, a bush along with the roller is fitted inside both the plates then a pin is passed through both the end of the roller to fasten it. The rollers are free to rotate inside the bush so that when it contacts between the sprocket the wear and tear would be minimized. Generally, it is made of steel. It is soundless and wears less as compared to the block chains and used in any circumstances and this type chains gave much service area if proper lubrication is maintained.

c) **Silent Chain or Inverted Tooth Chain:** The operation of this type of chain is quite silent and can be used in high-speed power transmissions areas. This type of chain does not have any roller; it is accurately shaped that it can easily sit over the sprocket. However, lubrication is an important factor for this type of chain otherwise it will wear and tear out.

TERMINOLOGY

- **Sprocket:** The chain is fitted on wheels are called sprockets. A sprocket has projected teeth which fit into the corresponding recesses in the chain.
- **Pitch:** It is the distance between the hinges centre of two adjacent links.
- **Pitch Circle:** A circle is drawn through the hinge centre, when the chain is wrapped around the sprockets.
- **Pitch Circle Diameter (PCD):** The diameter of pitch circle is called as pitch circle diameter.

2.6 GEAR DRIVE - SIMPLE, COMPOUND, REVERTED AND EPICYCLIC GEAR TRAINS (SIMPLE NUMERICALS)

GEARS DRIVE

Gears are the tooth wheels used to transmit power between two shafts, when the centre distance between them is very small. Gears are generally used for one of four different reasons:

- To increase or decrease the speed of rotation
- To change the amount of force or torque;
- To move rotational motion to a different axis
- To reverse the direction of rotation.

Gears are compact, positive-engagement, power transmission elements capable of changing the amount of force or torque. The gears can be classified according to:

- The position of shaft axes
- The peripheral velocity
- The type of gears
- The teeth position

According to the position of shaft axes:

Gears may be classified according to the relative position of the axes of revolution. The axes may be:

- Parallel shafts where the angle between driving and driven shaft is 0 degree. Examples include spur gears, single and double helical gears.
- Intersecting shafts where there is some angle between driving and driven shaft. Examples include bevel and miter gear.
- Non-intersecting and non-parallel shafts where the shafts are not coplanar. Examples include the hypoid and worm gear.

According to peripheral velocity:

Gears can be classified as:

- Low velocity type, if their peripheral velocity lies in the range of 1 to 3 m/sec.
- Medium velocity type, if their peripheral velocity lies in the range of 3 to 15 m/sec.
- High velocity type, if their peripheral velocity exceeds 15 m/sec.

According to type of gears:

Gears can be classified as external gears, internal gears, and rack and pinion.

- External gears mesh externally - the bigger one is called "gear" and the smaller one is called "pinion".
- Internal gears mesh internally - the larger one is called "annular" gear and the smaller one is called "pinion".
- Rack and pinion type – converts rotary to linear motion or vice versa. There is a straight line gear called "rack" on which a small rotary gear called "pinion" moves.

According to teeth position:

Gears are classified as straight, inclined and curved.

- Straight gear teeth are those where the teeth axis is parallel to the shaft axis.
- Inclined gear teeth are those where the teeth axis is at some angle.
- Curve gear teeth are curved on the rim's surface.

TYPE OF GEARS

SPUR GEARS:

Spur gears are used to transmit power between two parallel shafts. The teeth on these gears are cut straight and are parallel to the shafts to which they are attached.

Spur Gears Characteristics:

- Simplest and most economical type of gear to manufacture.
- Speed ratios of up to 8 (in extreme cases up to 20) for one step (single reduction) design; up to 45 for two step design; and up to 200 for three-step design.

Limitations:
- Not suitable when a direction change between the two shafts is required.
- Produce noise because the contact occurs over the full face width of the mating teeth instantaneously.

HELICAL GEARS

Helical gears resemble spur gears, but the teeth are cut at an angle rather than parallel to the shaft axis like on spur gears. The angle that the helical gear tooth is on is referred to as the helix angle. The angle of helix depends upon the condition of the shaft design and relative position of the shafts. To ensure that the gears run smoothly, the helix angle should be such that one end of the gear tooth remains in contact until the opposite end of the following gear tooth has found a contact. For parallel shafts, the helix angle should not exceed 20 degrees to avoid excessive end thrust.

Characteristics:
- The longer teeth cause helical gears to have the following differences from spur gears of the same size:
- Tooth strength is greater because the teeth are longer than the teeth of spur gear of equivalent pitch diameter.
- Can carry higher loads than can spur gears because of greater surface contact on the teeth.
- Can be used to connect parallel shafts as well as non-parallel, non-intersecting shafts.
- Quieter even at higher speed and are durable.
- Limitations:
- Gears in mesh produce thrust forces in the axial directions.
- Expensive compared to spur gears.

BEVEL GEARS:

A bevel gear is shaped like a section of a cone and primarily used to transfer power between intersecting shafts at right angles. The teeth of a bevel gear may be straight or spiral. Straight gear is preferred for peripheral speeds up to 1000 feet per minute; above that they tend to be noisy.

Characteristics:

- Designed for the efficient transmission of power and motion between intersecting shafts. A good example of bevel gears is seen as the main mechanism for a hand drill. As the handle of the drill is turned in a vertical direction, the bevel gears change the rotation of the chuck to a horizontal rotation.
- Permit a minor adjustment during assembly and allow for some displacement due to deflection under operating loads without concentrating the load on the end of the tooth.

MITTER GEARS:

Mitter gears are identical to bevel gears with the exception that both gears always have the same number of teeth.

Characteristics:

- They provide a steady ratio; other characteristics are similar to bevel gears.
- They are used as important parts of conveyors, elevators and kilns.
- Limitations
- Gear ration is always 1 to 1 and therefore not used when an application calls for a change of speed.

WORM GEARS:

Worm gears are used to transmit power between two shafts that are at right angles to each other and are non-intersecting. Worm gears are special gears that

resemble screws, and can be used to drive spur gears or helical gears. Worm gearing is essentially a special form of helical gearing in which the teeth have line contact and the axes of the driving and driven shafts are usually at right angles and do not intersect.

Characteristics:

- In the meshing of two external gears, rotation goes in the opposite direction. In the meshing of an internal gear with an external gear the rotation goes in the same direction.
- The meshing arrangement enables a greater load carrying capacity with improved safety (since meshing teeth are enclosed) compared to equivalent external gears.
- Shaft axes remain parallel and enable a compact reduction with rotation in the same sense. Internal gears are not widely available as standard.
- When they are used with the pinion, more teeth carry the load that is evenly distributed. The even distribution decreases the pressure intensity and increases the life of the gear.
- Allows compact design since the center distance is less than for external gears. Used in planetary gears to produce large reduction ratios.
- Provides good surface endurance due to a convex profile surface working against a concave surface. Applications:
- Planetary gear drive of high reduction ratios, clutches, etc.
- Limitations:
- Housing and bearing supports are more complicated because the external gear nests within the internal gear.
- Low ratios are unsuitable and in many cases impossible because of interferences.
- Fabrication is difficult and usually special tooling is required.

GEAR TRAINS

Gear train is a power transmission system made up of two or more gears. The gear to which the force is first applied is called the driver and the final gear on the train to which the force is transmitted is called the driven gear. Any gears between the driver and the driven gears are called the idlers. Conventionally, the smaller gear is the **Pinion** and the larger one is the **Gear**. In most applications, the pinion is the driver; this reduces speed but increases torque.

Types of gear trains:

- Simple gear train
- Compound gear train
- Planetary gear train

Simple Gear Train - Simple gear trains have only one gear per shaft. The simple gear train is used where there is a large distance to be covered between the input shaft and the output shaft.
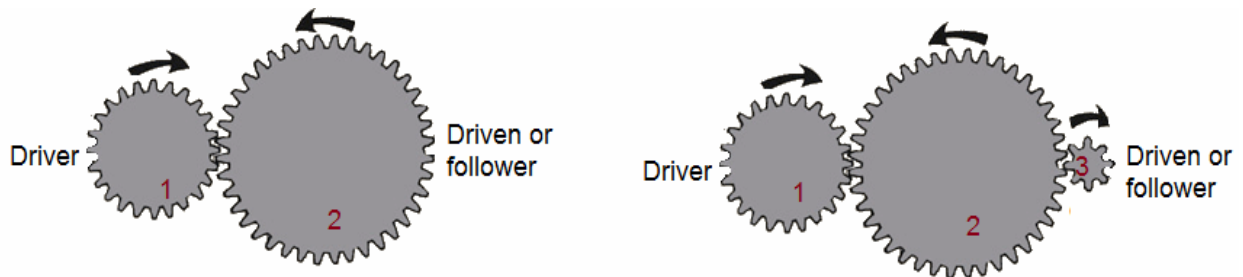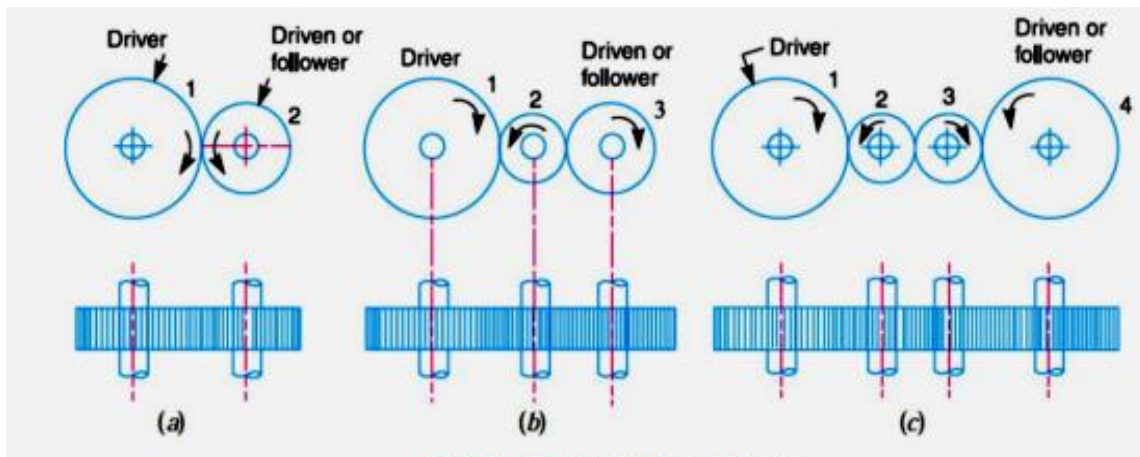
Fig. 13.1. Simple gear train.

Let N_1 = Speed of gear 1 (or driver) in r.p.m.,

N_2 = Speed of gear 2 (or driven or follower) in r.p.m.,

T_1 = Number of teeth on gear 1, and

T_2 = Number of teeth on gear 2.

Since the speed ratio (or velocity ratio) of gear train is the ratio of the speed of the driver to the speed of the driven or follower and ratio of speeds of any pair of gears in mesh is the inverse of their number of teeth, therefore

$$\text{Speed ratio} = \frac{N_1}{N_2} = \frac{T_2}{T_1}$$

It may be noted that ratio of the speed of the driven or follower to the speed of the driver is known as train value of the gear train. Mathematically, From above, we see that the train value is the reciprocal of speed ratio.

$$\text{Train value} = \frac{N_2}{N_1} = \frac{T_1}{T_2}$$

Compound Gear Train - In a compound gear train at least one of the shafts in the train must hold two gears. Compound gear trains are used when large

changes in speed or power output are needed and there is only a small space between the input and output shafts.
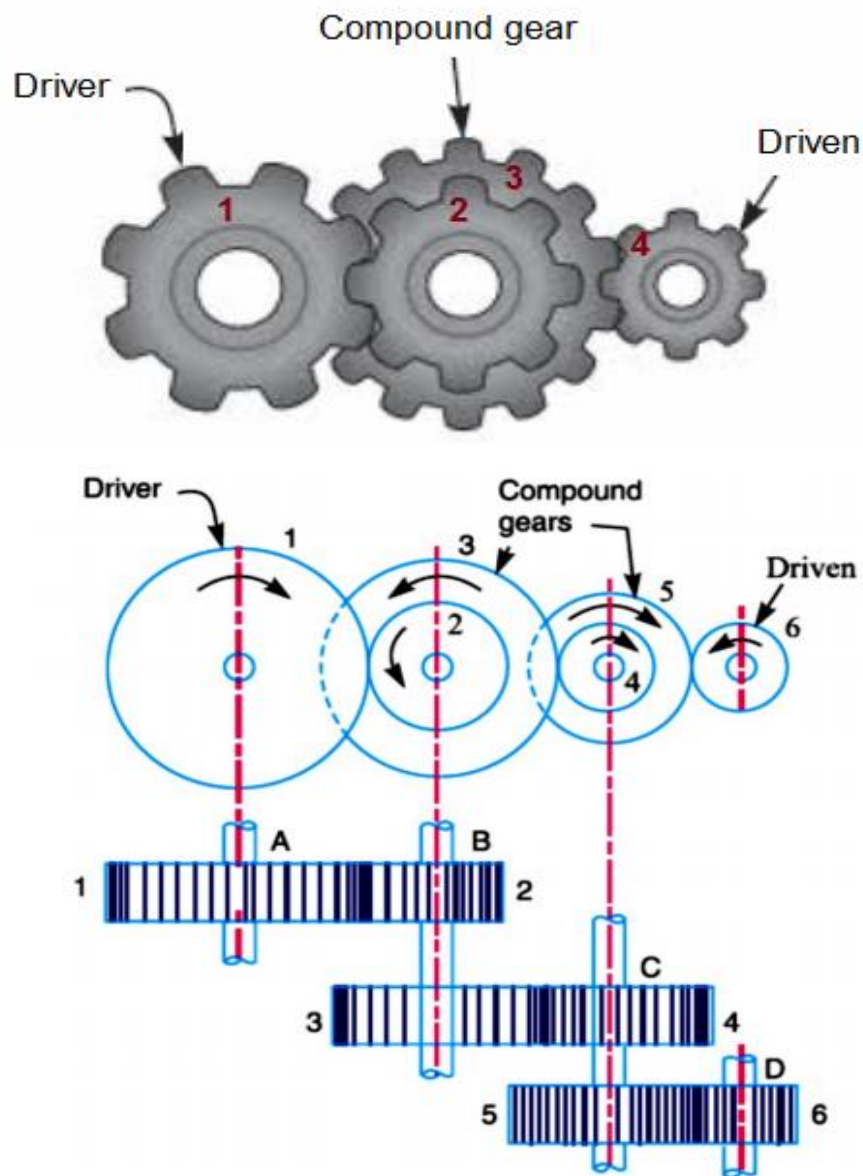
In a compound train of gears, as shown in Fig. 13.2, the gear 1 is the driving gear mounted on shaft A, gears 2 and 3 are compound gears which are mounted on shaft B. The gears 4 and 5 are also compound gears which are mounted on shaft C and the gear 6 is the driven gear mounted on shaft D.

Let

N_1 = Speed of driving gear 1,

T_1 = Number of teeth on driving gear 1,

N_2, N_3 ..., N_6 = Speed of respective gears in r.p.m., and

T2 ,T3..., T6 = Number of teeth on respective gears.

Since gear 1 is in mesh with gear 2, therefore its speed ratio is

$$\frac{N_1}{N_2} = \frac{T_2}{T_1} \quad \ldots\ldots\ldots\ldots(i)$$

Similarly, for gears 3 and 4, speed ratio is

$$\frac{N_3}{N_4} = \frac{T_4}{T_3} \quad \ldots\ldots\ldots\ldots(ii)$$

and for gears 5 and 6, speed ratio is

$$\frac{N_5}{N_6} = \frac{T_6}{T_5} \quad \ldots\ldots\ldots\ldots(iii)$$

The speed ratio of compound gear train is obtained by multiplying the equations *(i)*, *(ii)* and *(iii)*,

$$\frac{N_1}{N_2} \times \frac{N_3}{N_4} \times \frac{N_5}{N_6} = \frac{T_2}{T_1} \times \frac{T_4}{T_3} \times \frac{T_6}{T_5}$$

Since gears 2 and 3 are mounted on one shaft B, therefore $N_2 = N_3$. Similarly gears 4 and 5 are mounted on shaft C, therefore $N_4 = N_5$.

$$\frac{N_1}{N_6} = \frac{T_2 \times T_4 \times T_6}{T_1 \times T_3 \times T_5}$$

i.e

$$Speed\ ratio = \frac{Speed\ of\ the\ first\ driver}{Speed\ of\ last\ driven} = \frac{Product\ of\ the\ number\ of\ teeth\ on\ the\ drivens}{Product\ of\ the\ number\ of\ teeth\ on\ the\ drivers}$$

and $Train\ value = \dfrac{Speed\ of\ last\ driven}{Speed\ of\ the\ first\ driver} = \dfrac{Product\ of\ the\ number\ of\ teeth\ on\ the\ drivers}{Product\ of\ the\ number\ of\ teeth\ on\ the\ drivens}$

The advantage of a compound train over a simple gear train is that a much larger speed reduction from the first shaft to the last shaft can be obtained with small gears. If a simple gear train is used to give a large speed reduction, the last gear has to be very large. Usually for a speed reduction in excess of 7 to 1, a simple train is not used and a compound train or worm gearing is employed.

Note: The gears which mesh must have the same circular pitch or module. Thus gears 1 and 2 must have the same module as they mesh together. Similarly gears 3 and 4, and gears 5 and 6 must have the same module.

Planetary Gear Train - A planetary transmission system or it is also known as epicyclic system. It consists normally of a centrally pivoted sun gear, a ring gear and several planet gears which rotate between these. This assembly concept explains the term planetary transmission, as the planet gears rotate around the sun gear as in the astronomical sense the planets rotate around our sun.
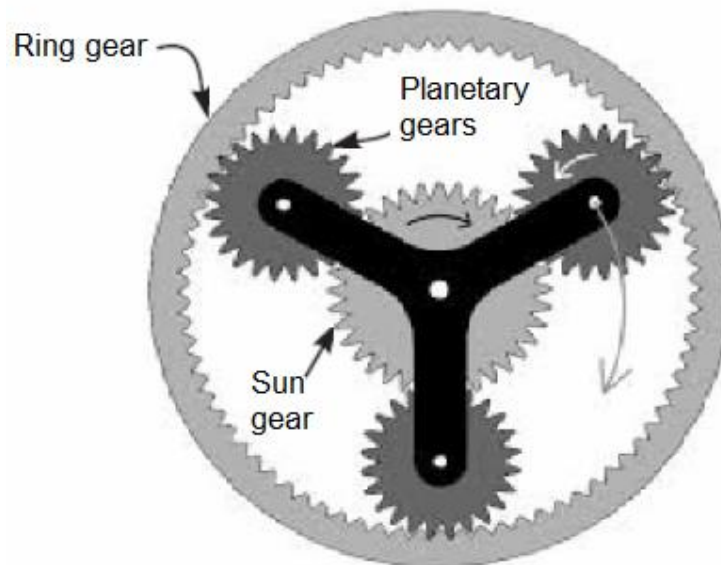
Reverted Gear Train: When the axes of the first gear (i.e. first driver) and the last gear (i.e. last driven or follower) are co-axial, then the gear train is known as reverted gear train. We see that gear 1 (i.e. first driver) drives the gear 2 (i.e. first driven or follower) in the opposite direction. Since the gears 2 and 3 are mounted on the same shaft, therefore they form a compound gear and the gear 3 will rotate in the same direction as that of gear 2. The gear 3 (which is now the second driver) drives the gear 4 (i.e. the last driven or follower) in the same direction as that of gear 1. Thus we see that in a reverted gear train, the motion of the first gear and the last gear is like.

Let

T1 = Number of teeth on gear 1,

r1 = Pitch circle radius of gear 1,

N1 = Speed of gear 1 in r.p.m.

Similarly,

T2, T3, T4 = Number of teeth on respective gears,

r2, r3, r4 = Pitch circle radii of respective gears, and

N2, N3, N4 = Speed of respective gears in r.p.m.
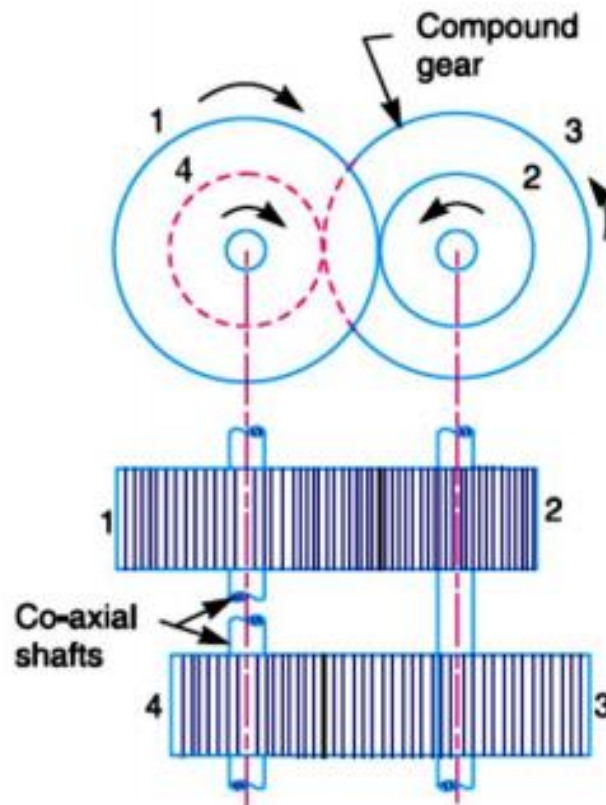
Since the distance between the centers of the shafts of gears 1 and 2 as well as gears 3 and 4 is same, therefore

r1 + r2 = r3 + r4   ... (i)

Also, the circular pitch or module of all the gears is assumed to be same, therefore number of teeth on each gear is directly proportional to its circumference or radius

We know that circular pitch,

$$p_c = \frac{2\pi r}{T} = \pi m \qquad \text{or} \qquad r = \frac{m.T}{2}, \text{ where } m \text{ is the module.}$$

$$\therefore \qquad r_1 = \frac{m.T_1}{2} \; ; \; r_2 = \frac{m.T_2}{2} \; ; \; r_3 = \frac{m.T_3}{2} \; ; \; r_4 = \frac{m.T_4}{2}$$

Now from equation (i),

$$\frac{m.T_1}{2} + \frac{m.T_2}{2} = \frac{m.T_3}{2} + \frac{m.T_4}{2}$$

$$\therefore T_1 + T_2 = T_3 + T_4 \qquad \qquad ...(ii)$$

$$Speed\ Ratio = \frac{Product\ of\ the\ number\ of\ teeth\ on\ the\ drivens}{Product\ of\ the\ number\ of\ teeth\ on\ the\ drivers}$$

or

$$\frac{N_1}{N_4} = \frac{T_2 \times T_4}{T_1 \times T_3} \qquad \qquad ...(iii)$$

From equations (i), (ii) and (iii), we can determine the number of teeth on each gear for the given centre distance, speed ratio and module only when the number of teeth on one gear is chosen arbitrarily. The reverted gear trains are used in automotive transmissions, lathe back gears, industrial speed reducers, and in clocks (where the minute and hour hand shafts are co-axial).

GEAR RATIO

The ratio of number of teeth on gear to that on the pinion is called gear ratio.

$$\textbf{Gear Ratio} = \frac{\textbf{T}}{\textbf{t}}$$

Where,

T= Number of teeth on gear

t = Number of teeth on pinion

2.7 RELATIVE ADVANTAGES AND DISADVANTAGES OF VARIOUS DRIVES

ADVANTAGES AND DISADVANTAGES OF BELT DRIVE

Advantages:

- Belt drives are simple are economical.
- They don't need parallel shafts.
- Belts drives are provided with overload and jam protection.
- Noise and vibration are damped out.
- Machinery life is increased because load fluctuations are shock-absorbed.
- They are lubrication-free.
- They require less maintenance cost.
- Belt drives are highly efficient in use (up to 98%, usually 95%).
- They are very economical when the distance between shafts is very large.

Disadvantages:

- In Belt drives, angular velocity ratio is not necessarily constant or equal to the ratio of pulley diameters, because of slipping and stretching.
- Heat buildup occurs. Speed is limited to usually 35 meters per second. Power transmission is limited to 370 kilowatts.
- Operating temperatures are usually restricted to –35 to 85°C.
- Some adjustment of center distance or use of an idler pulley is necessary for wearing and stretching of belt drive compensation.

ADVANTAGES AND DISADVANTAGES OF CHAIN DRIVE

Advantages:

- They can be used for both long and short distances
- A number of shafts and be driven from a single chain
- They are compact and have small overall dimensions

- They do not present fire hazard

- Temperature and environmental conditions do not affect their working

- They do not require initial tension

- They have very high efficiency (up to 96%)

- They do not slip

- They are easier to install

- They can withstand abrasive conditions

- They can operate in wet conditions

Disadvantages:

- They can't be used where slip is the system requirement

- They require precise alignment compared to belt drives

- They require frequent lubrication

- They have less load capacity compared with gear drives

- Their operation is noisy and can cause vibrations

- They are not suitable for non-parallel shafts

- They require housing

- They require adjustments for slack like tensioning device

ADVANTAGES AND DISADVANTAGES OF GEAR DRIVE

Advantages:

- By using gear trains, large velocity ratio can be obtained with minimum space.

- Gears are mechanically strong, so higher loads can be lifted.

- Gears are used for transmission of large H.F.

- They are used for transmitting motion small centre distance of shafts

- They are used for large reduction in speed and for transmission of torque.

- Gears require only lubrication, hence less maintenance is required.

- Using gear systems, we can transmit motion between non-parallel intersecting shafts.

- They are used for positive drive, so its velocity ratio remains constant.

- They have long life, so the gear system is very compact.

Disadvantages:

- They are not suitable for large velocities.

- They are not suitable for transmitting motion over a large distance.

- Due to the engagement of toothed wheel of gears, some part of machine may get permanently damaged in case of excessive loading.

- They have no flexibility.

- Gear operation is noisy.

CHAPTER-3

FLYWHEEL

3.1 PRINCIPLE & APPLICATIONS OF FLYWHEEL

PRINCIPAL OF FLYWHEEL

A flywheel is a mechanical device specifically designed to efficiently store rotational energy. Flywheels resist changes in rotational speed by their moment of inertia. A flywheel is fitted on the crank shaft of engine. As the flywheel rotates with the crank shaft, it absorbs and releases energy which is proportional to the square of its rotational speed. Flywheel acts as an energy reservoir. Whenever the energy available is in excess than the required one, it absorbs the energy an when the available energy is less than the required one, it releases the energy. When the flywheel absorbs energy its speed increases and when it releases energy its speed decreases, this occurs at least once during a cycle.
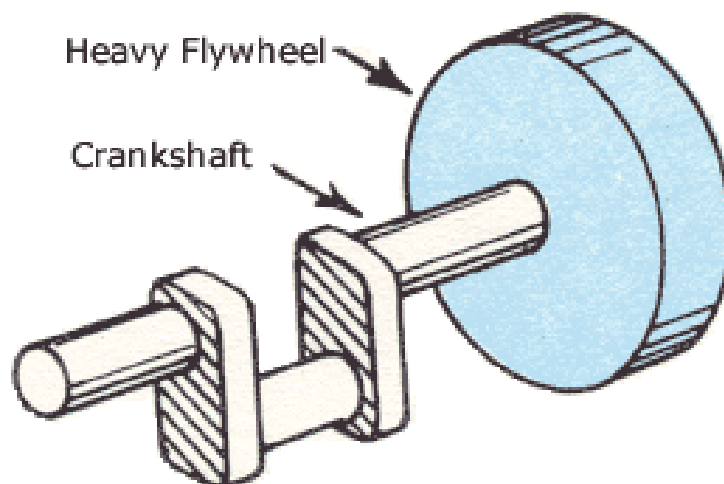
APPLICATION OF FLYWHEEL

Flywheels are often used to provide continuous power output in systems where the energy source is not continuous. For example, a flywheel is used to smooth fast angular velocity fluctuations of the crankshaft in a reciprocating engine. In this case, a crankshaft flywheel stores energy when torque is exerted on it by a firing piston, and returns it to the piston to compress a fresh charge of air and fuel. A flywheel may also be used to supply intermittent pulses of energy at power levels that exceed the abilities of its energy source. This is achieved by accumulating energy in the flywheel over a period of time, at a rate that is compatible with the energy source, and then releasing energy at a much higher rate over a relatively short time when it is needed. For example, flywheels are used in power hammers and riveting machines.

3.2 TURNING – MOMENT DIAGRAM OF FLYWHEEL FOR DIFFERENT ENGINES

It is the graphical representation of the turning moment or crank-effort for various positions of the crank. It is plotted on Cartesian co-ordinates, in which the turning moment is taken as the ordinate and crank angle as abscissa. The turning moment diagram (also known as crank- effort diagram) is the graphical representation of the turning moment or crank-effort for various positions of the crank. It is plotted on Cartesian co-ordinates, in which the turning moment is taken as the ordinate and crank angle as abscissa.
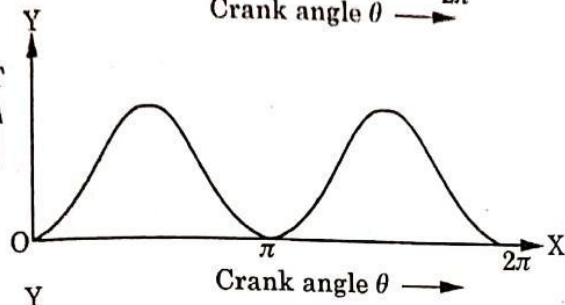
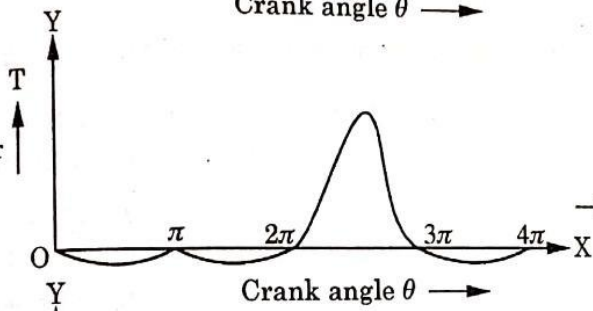Following figures show the turning moment diagrams of different engines.

Fly...

Single cylinder single acting steam engine

Crank angle θ —→
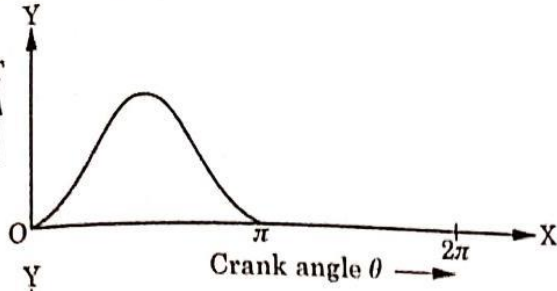
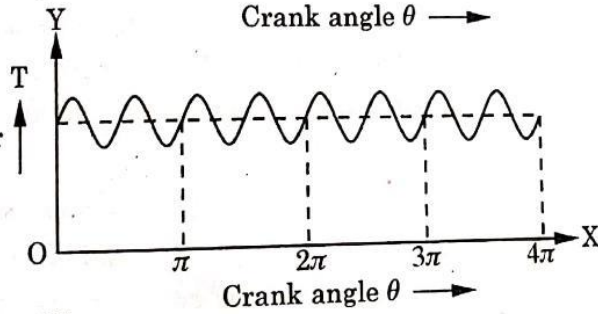Single cylinder double acting steam engine

Crank angle θ —→

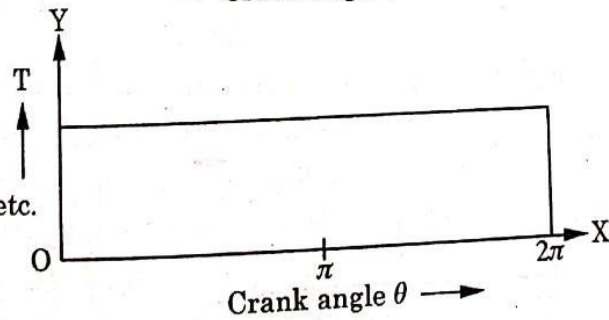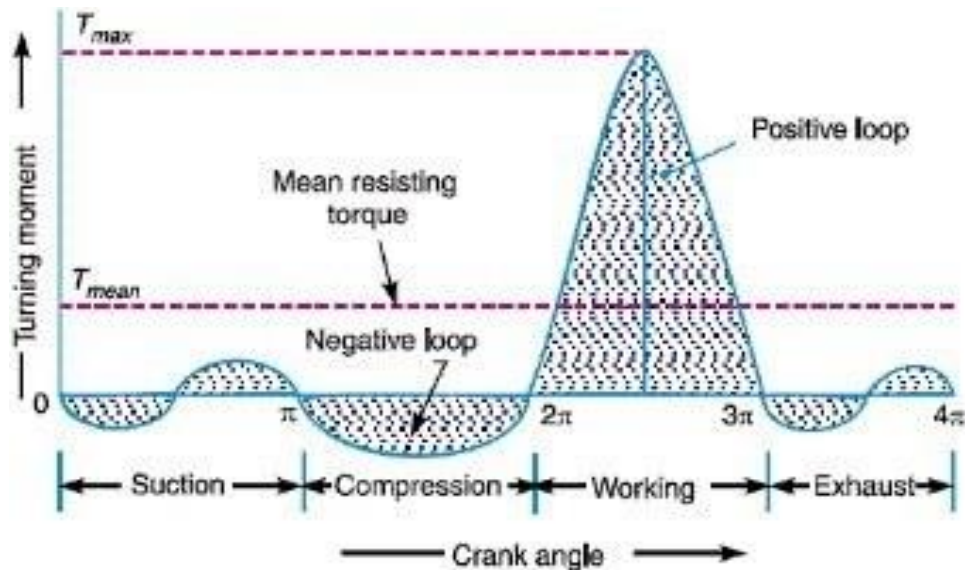Four stroke single cylinder I.C. engine

Crank angle θ —→

$0 - \pi$ = Suction stroke
$\pi - 2\pi$ = Compression stroke
$2\pi - 3\pi$ = Power stroke
$3\pi - 4\pi$ = Exhaust stroke

Eight cylinder four stroke I.C. engine

Crank angle θ —→

Turbine, electric motor etc.

Crank angle θ —→

Turning moment diagram for four stroke single cylinder I.C. Engine

3.3 FLUCTUATION OF SPEED AND FLUCTUATION OF ENERGY

FLUCTUATION OF SPEED: The difference between the maximum and minimum speeds during a cycle is called the maximum fluctuation of speed.

$$\text{Fluctuation of speed} = (N_1 - N_2) \text{ rpm}$$

Where, N_1 - maximum speed,

N_2 - minimum speed

FLUCTUATION OF ENERGY: It is the difference between the maximum and minimum energy of Flywheel.

$$\text{Fluctuation of energy} = I(\omega_1{}^2 - \omega_2{}^2) \text{ in N-m or J}$$

$$\text{Maximum energy of Flywheel} = I\omega_1{}^2$$

$$\text{Minimum energy of Flywheel} = I\omega_2{}^2$$

$$I - \text{moment of inertia of flywheel} = mk^2$$

Where,   m – mass of the flywheel in kg

k - radius of gyration of flywheel,

ω_1 – Maximum Angular velocity, rad/sec

ω_2 – Minimum Angular velocity, rad/sec

3.4 COEFFICIENT OF FLUCTUATION OF SPEED AND COEFFICIENT OF FLUCTUATION OF ENERGY

COEFFICIENT OF FLUCTUATION OF SPEED

It is the ratio of maximum fluctuation of speed to the mean speed. The maximum fluctuation of speed is difference between maximum and minimum speeds during a cycle.

$$\text{Coefficient of fluctuation of speed} = \frac{N1 - N2}{N}$$

Where,

N1 and N2 = Maximum and Minimum speeds in r.p.m during the cycle,

N = Mean speed in r.p.m. = (N1+N2)/2

COEFFICIENT OF FLUCTUATION OF ENERGY

It is the ratio of maximum fluctuation energy to the work done per cycle.

$$C_e = \Delta E / E$$

$$E = \frac{1}{2} I \, \omega^2_{mean}$$

ΔE = Maximum Fluctuation of energy as speed changes from ω_{max} to ω_{min}

CHAPTER-4

GOVERNOR

4.1 FUNCTION OF A GOVERNOR, COMPARISON OF FLYWHEEL AND GOVERNOR

FUNCTION OF A GOVERNOR

The function of the governor is to adjust the fuel supply automatically with the change of the external load of the diesel engine within the required speed range of the diesel engine, so as to keep the diesel engine speed basically stable. When the load on an engine increases, its speed decreases, therefore it becomes necessary to increases the supply of working fluid. On the other hand, when the load on the engine decreases, its speed increases and thus less working fluid is required. One of the types of governor is centrifugal governor which uses effect of centrifugal force on rotating weights driven by the machine output shaft to regulate its speed by altering the input amount of fuel.

Types of Governors

There are basically two types of governors:

- Centrifugal governors
- Inertia governor

The centrifugal governors can be further classified as follows:

1. **Pendulum type—Watt governor**

2. **Loaded type –**

   There are two kinds of loaded governors:

   (a) Dead weight type

   (b) Spring-loaded type

(a) Dead weight type:

        (i) Porter governor

        (ii) Proell governor

(b) Spring-loaded type

        (i) Hartnell governor

        (ii) Hartung governor

   (iii) Wilson–Hartnell governor

        (iv) Pickering governor

COMPARISON OF FLYWHEEL AND GOVERNOR

	FLYWHEEL	GOVERNOR
1	Flywheel is used to prevent fluctuation of energy and does not maintain a constant speed.	Governor is used to maintain the constant speed, whenever there are changes in load.
2	Generally, flywheel is a heavy part of the machine.	Governor is a lighter in weight than flywheel.
3	It is a rotating component.	It is a non-rotating component.
4	Energy generated due to flywheel is directly proportional to the square of its angular speed.	There is no such proportionality or equation in a governor.
5	As there are energy variations in engines, a flywheel results in a maintaining a constant speed of crankshaft in each cycle by supply the required energy to the preparatory stokes (other than power stroke).	As the load on engine increases, the speed decreases. This increases the flow of fuel for regulating constant speed.

4.2 SIMPLE DESCRIPTION AND WORKING OF WATT, PORTER AND HARTNEL GOVERNOR (SIMPLE NUMERICAL BASED ON WATT AND PORTER GOVERNOR)

WATT GOVERNOR

Watt governor is the simplest and gravity controlled form of the centrifugal governors. It consists of two fly balls attached to the sleeve of negligible mass. The upper sides of arms are pivoted so that its balls can move upward and downward as they revolve with a vertical spindle. The engine drives the spindle through bevel gears. The lower arms are connected to the sleeves. The sleeve is keyed to the spindle in such a way that it revolves with the spindle. At the same time, it can slide up and down according to the spindle speed. Two stoppers are provided at the bottom and top of the spindle to limit the movement sleeve.
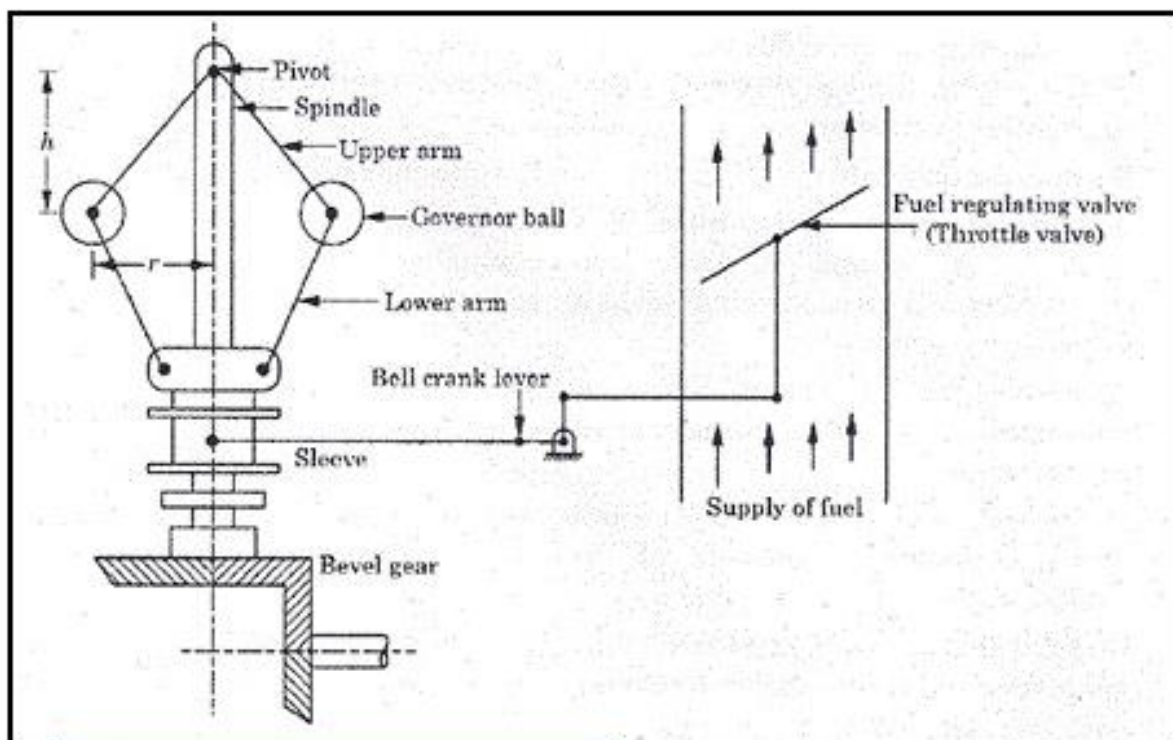
When the load on the engine decreases, the speed of the engine and then the angular velocity of the governor spindle increase. The centrifugal force on the ball increase; that tends balls move outward and sleeve move upward. The upward movement of the sleeve actuates a mechanism that operates the throttle valve at the end of bell crank lever to decrease the fuel supply. The power output is reduced. When the speed of the engine decreases as the load on the engine increase, the centrifugal force decreases. The result is that the inward movement fly-balls and downward movement of the sleeve. The movement causes a wide opening of the throttle valve. The increase in the fuel supply also increases engine speed.

PORTER GOVERNOR

Porter Governor is modification of Watt Governor with central load attached to the sleeve. This load moves up and down the central spindle. The additional force increases the speed of revolution required to enable the balls to rise to any predetermined level.

Construction:

A Porter governor has two fly balls which are attached to the arms of the porter governor. These two arms are pivoted to the top of the spindle. This spindle is driven by the engine. The parts of the arms just above the fly balls are connected to the central sleeve. This sleeve moves up and down according to the movement of the balls. There are stoppers place in the spindle to limit the vertical movement of the spindle. This sleeve is carries a heavy central load. The movement of the sleeve controls opening and closing of the throttle valve.

Working:

When the load of engine decreases, there will be sudden increase in speed of engine and spindle speed will also increase. As the spindle speed increases two fly balls also start rotating around the spindle fast. The centrifugal force will

push the balls outward making the balls move in upward direction. Since the ball moves upward , the arms also moves upward and the sleeve connected to the lower part of the arm also moves upward. Upward movement of sleeve actuates the throttle valve via a mechanism connected to the sleeve to decrease the fuel supply to the engine. The decreases in fuel supply decrease the speed. Hence   speed   is   maintained.
In the other case, when the load of the engine increases speed of the engine decreases. As the engine speed decreases, speed of the spindle also decreases and centrifugal force in the balls. Hence the balls come down with the arms. As the arms come down, the sleeve connected to the arm also comes down and it actuates the throttle valve which increases the fuel supply. Due to increase in fuel supply, speed of engine also increases.

PORTER GOVERNOR
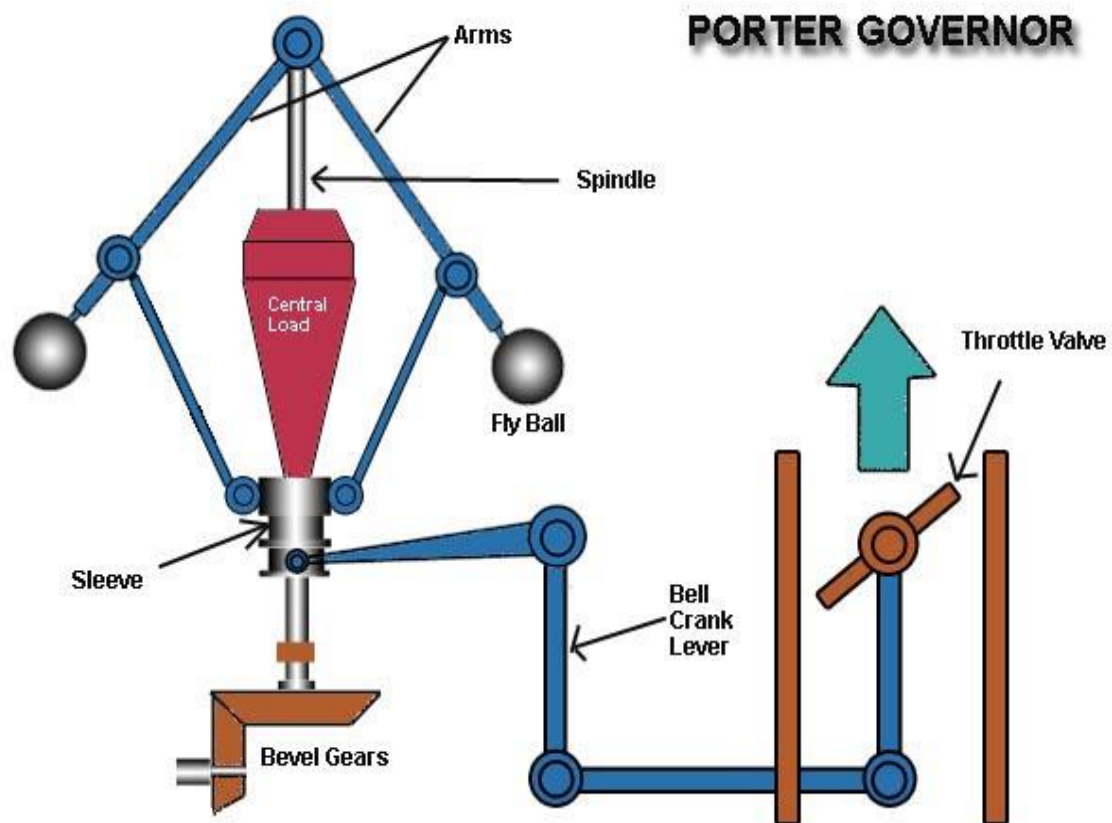
HARTNEL GOVERNOR

Mr. Hartnell designed his governor circa 1875. It is a Spring-loaded Governor. The function of a governor is to regulate the mean speed of an engine, when there are variations in the load e.g. when the load on an engine increases, its speed decreases, therefore it becomes necessary to increase the supply of working fluid. On the other hand, when the load on the engine decreases, its speed increases, and thus less working fluid is required. The governor automatically controls the supply of working fluid to the engine with the varying load conditions and keeps the mean speed within certain limits.

Construction:

It is a Spring-loaded Governor. It has two bell crank levers carrying the fly ball at one end and roller attached to the other end, the function of spring is to provide the counterforce which acts against centrifugal force. The spring and shaft are enclosed inside a casing. The sleeve is pressed against the spring when the centrifugal force on the balls increases. Due to spring return nature this governor can be mounded in a horizontal, inverted, (inclined) position.

Parts of Hartnell Governor:

The following are the main parts of the Hartnell Governor.

- **Frame:** This is provided with a spring inside which protects and supports it and the frame is connected to the bell crank lever which rotates along with the lever.
- **Balls:** These are placed on the bell crank lever which is done with some specified weight.
- **Bell crank lever:** This is used to rotate along with the balls provided on it and it is connected to the frame to rotate it.
- **Nut:** This is proved on the top of the frame, which used to adjust the force on the sleeve.

- **Spring:** This is used to apply the pressure on the sleeve whenever required and this helps to push the sleeve downwards when it raised more than the required.
- **Collar:** This provided on the top of the sleeve to support the spring on it.
- **Sleeve:** This used to move in an upward and downward direction according to the rotation of the bell crank lever attached to it and this helps to flow the required amount of fluid into the engine when it raised above from the bottom point.

Working:

Hartnell governor is a spring controlled centrifugal governor, in which a spring controls the movement of the ball and hence the sleeve. Above fig shows a Hartnell governor. It consists of a frame /casing, in which a pre com pressed helical spring is housed. The casing and spring can rotate about the spindle axis. The spring applies a downward force on the sleeve through an adjustable collar. The spring force can be adjusted by a nut provided. Two bell crank levers are pivoted at O,O' to the frame, each carrying a ball at one end a roller at another end. The roller fits into the grooves of the sleeve.

The sleeve moves up and down depending on the governor's speed. When the speed of the governor/ engine increases, the ball tends to fly outward from the axis of the governor, but the ball's movement is constrained. The bell crank lever moves on a pivot, roller end of lever lifts the sleeve upward against the spring force. This movement transferred to the throttle valve through a suitable mechanism, the result is low fuel supply and decreasing speed. When speed decreases the sleeve moves downward, and throttle opens to more fuel supply; which results in increasing speed.
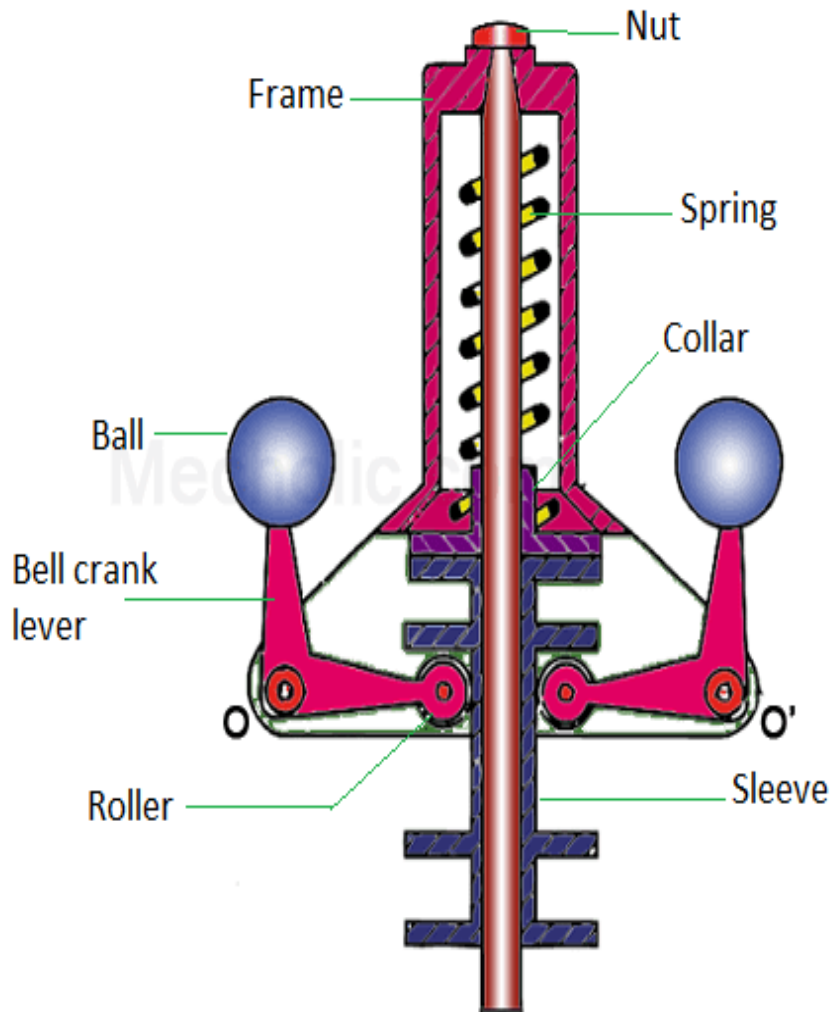
Advantages of Hartnell governor:

- It can be operated at very high speed
- Smaller in size
- Very close regulation
- Pre-compression can be adjusted to give the required equilibrium speed

4.4 TERMINOLOGY USED IN GOVERNORS: HEIGHT, EQUILIBRIUM SPEED, HUNTING, ISOCHRONISMS, STABILITY, SENSITIVENESS OF A GOVERNOR

Height of Governor: It is the vertical distance from the centre of the ball to a point where the axes of the arms (or arms produced) intersect on the spindle axis. It is usually denoted by h.

Equilibrium Speed: It is the speed at which the governor balls, arms etc., are in complete equilibrium and the sleeve does not tend to move upwards or downwards.

Mean equilibrium speed: It is the speed at the mean position of the balls or the sleeve.

Maximum and minimum equilibrium speeds: The speeds at the maximum and minimum radius of rotation of the balls, without tending to move either way are known as maximum and minimum equilibrium speeds respectively.

Sleeve lift: It is the vertical distance which the sleeve travels due to change in equilibrium speed.

Hunting: It is a condition in which the speed of the engine controlled by the governor fluctuates continuously above and below the mean speed. It is caused by a governor which is too sensitive.

Isochronisms: A governor is said to be isochronous, when the equilibrium speed is constant for all radii of rotation of the balls, within the working range. An isochronous governor will be infinitely sensitive.

Stability: A governor is said to be stable if it brings the speed of engine to the required value and there is not much hunting. The ball masses occupy a definite position for each speed of the engine within the working range.

Sensitiveness: A governor is said to be sensitive when it readily respond to a small variation in speed. It may also be defined as the ratio of mean speed to the range of speed of governor.

$$Sensitiveness = \frac{Mean\ speed}{Range\ of\ speed}$$

$$Sensitiveness = \frac{N}{N1 - N2}$$

The stability and sensitiveness of a governor are two opposite characteristics.

CHAPTER-5

CAMS

5.1 DEFINITION AND FUNCTION OF CAM, DESCRIPTION OF DIFFERENT TYPES OF CAMS AND FOLLOWERS WITH SIMPLE LINE DIAGRAM.

DEFINITION AND FUNCTION OF A CAM

Cam is a mechanical element which is used to deliver the reciprocating or oscillating motion to another machine element known as follower. Cam and follower is a type of assembly, both works together in any machine element. Cam and followers are the example of higher pair due to the line contact between them. Some complicated motions are easily achieved by using cams and followers which are difficult to achieve. We can say that the cam and followers are the most important part of the mechanisms. The cams and follower combination is mostly used in internal combustion engines for operating inlet and exhaust valves, machine tools, printing control mechanism so on. They are generally manufactured by using die casting, milling and by punching. A cam is a rotating or sliding piece in a mechanical linkage that drives a mating component known as a follower. From a functional viewpoint, a cam-and-follower arrangement is very similar to the linkages. The cam accepts an input motion (rotary motion or linear motion) and imparts a resultant motion (linear motion or rotary motion) to a follower.

A cam and follower are a method of converting rotary motion into linear motion. The most well-known application is in an internal combustion engine,

where the cam and follower combination determines the opening times and duration of the valves. (via the pushrod and rocker arm).

Other applications might be on industrial machinery, regulating opening and closing of equipment for filling bottles or containers, toys (for example a "quacking duck" toy, where a cam can be used to move the duck's beak and make the quack noise), in fact, any application where you want a fairly short linear motion at a regular repeated interval. Change of circular motion to the translatory (linear) motion of simple harmonic type and vice-versa and can be done by slider-crank mechanism as discussed previously. But now the question arises, what to do when circular or rotary motion is to be changed into linear motion of complex nature or oscillatory motion. This job is well accomplished by a machine part of a mechanical member, known as cam.

DIFFERENT TYPES OF CAM AND FOLLOWER

Types of Cam:

There are two types of cams:

- **Radial or Disc cam**: In radial cam, cam profile (shape of cam) is design in such way by which follower move as per the shape of cam or cam profile in perpendicular direction to the cam axis. There is always surface contact between cam profile and follower by spring force or gravity force.

- **Cylindrical cam:** In cylindrical cam, follower axis and cam axis are parallel to each other. Cylindrical cam is cylindrical shaft on which profile made on cylindrical surface by machining. This profile is in the groove form and follower has surface contact with this groove. As cylindrical shaft or cylindrical cam is rotates follower move and movement of follower may be reciprocating or oscillating.
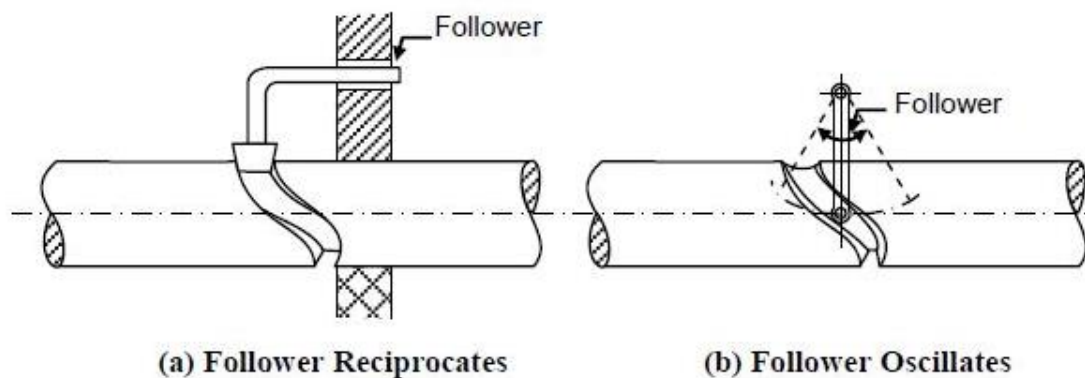
Radial or Disc cam

(a) Follower Reciprocates   (b) Follower Oscillates
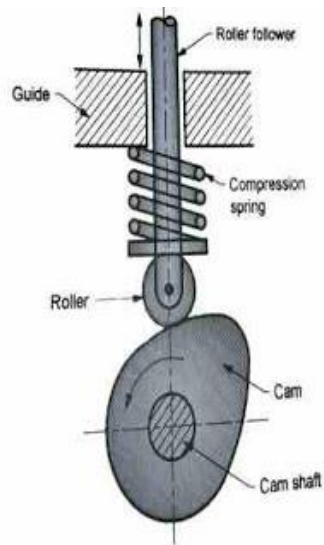
Figure 1.50 : Cylindrical Cams

cylindrical cam

Types of Followers:

Followers may be classified in three different ways:

    a) Depending upon the type of motion, i.e. reciprocating or oscillating

    b) Depending upon the axis of the motion, i.e. radial or offset.

   c) Depending upon the shape of their contacting end with the cam: Under this classification followers may be divided into three types:

- Knife-edge Follower
- Roller Follower
- Flat or Mushroom Follower

Knife-edge Follower: Knife-edge followers are generally, not used because of the obvious high rate of wear at the knife edge. However, cam of any shape can be worked with it. During working, considerable side thrust exists between the follower and the guide.
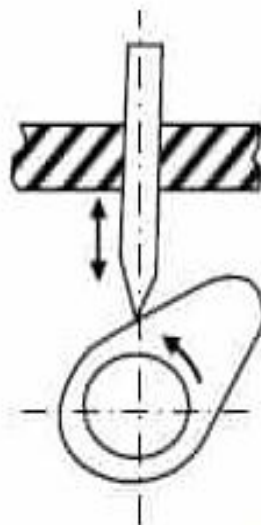

(a) Knife-edge Follower

Roller Follower: In place of a knife-edge, a roller is provided at the contacting end of the follower, hence, the name roller follower. Instead of sliding motion between the contacting surface of the follower and the cam, rolling motion takes place, with the result that the rate of wear is greatly reduced. In roller followers also, as in knife-edge follower, side thrust is exerted on the follower guide. Roller followers are extensively used in stationary gas and oil engines. They are also used in aircraft engines due to their limited wear at high cam velocity. While working on the concave surface of a cam the radius of the surface must be at least equal to the radius of the roller.

(b) Roller Follower
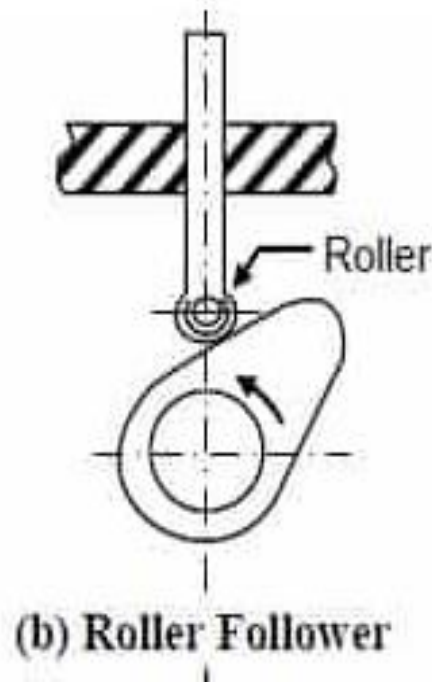
Flat or Mushroom Follower: At the name implies the contacting end of the follower is flat as shown. In mushroom followers there is no side thrust on the guide except that due to friction at the contact of the cam and the follower. No doubt that there will be sliding motion between the contacting surface of the follower and the cam but the wear can be considerably reduced by off-setting the axis of the followers. The off-setting provided causes the follower to rotate about its axis when the cam rotates. Flat face follower is used where the space is limited. That is why it is used to operate valves of automobile engines. Where sufficient space is available as in stationary gas and oil engines, roller follower is used as mentioned above. The flat-faced follower is generally preferred to the roller follower because of the compulsion of having to use a small diameter of the pin in the roller of the roller follower. In flat followers, high surface stresses are produced in the flat contacting surface. To minimize these stresses, a spherical shape is given to the flat end.
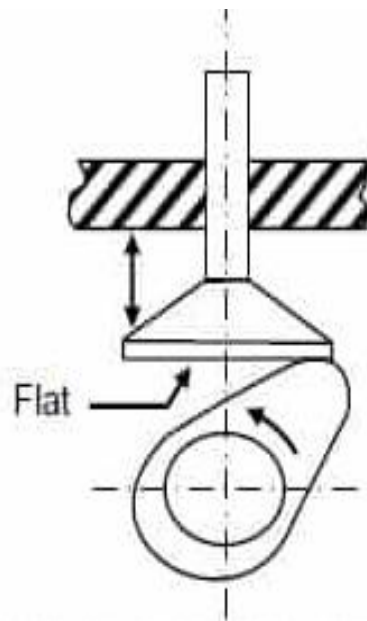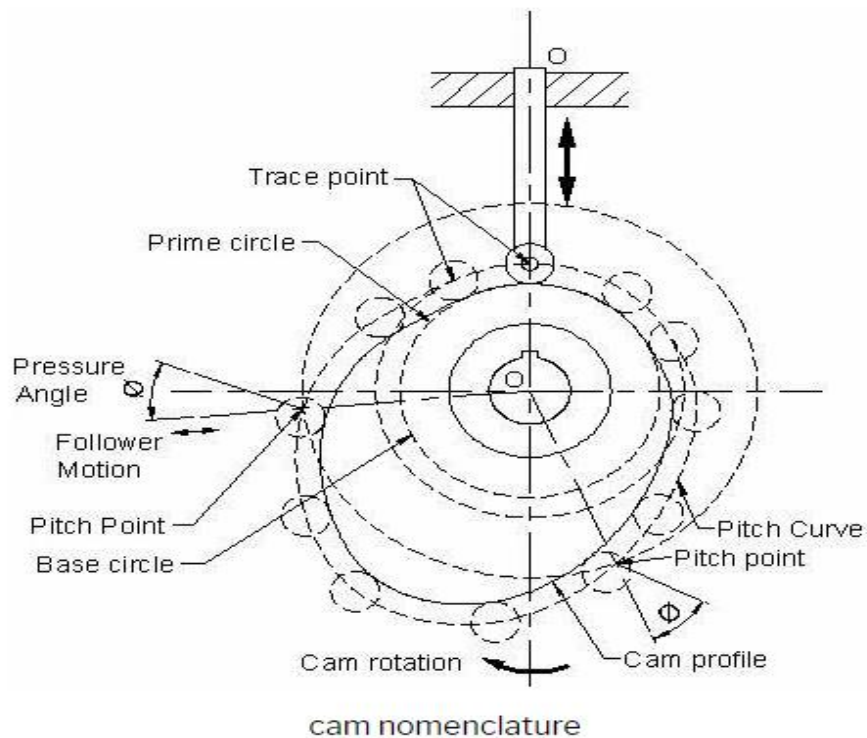
(c) Flat or Mushroom Follower

5.2 TERMINOLOGY OF CAM PROFILE

- **Cam profile**: Cam profile is outer surface of the disc cam.
- **Base circle:** Base circle is the smallest circle, drawn tangential to the cam profile.


cam nomenclature

- **Trace point**: Trace point is a point on the follower, trace point motion describes the movement of the follower.
- **Pitch curve:** Pitch curve is the path generated by the trace point as the follower is rotated about a stationery cam.
- **Prime circle**: Prime circle is the smallest circle that can be drawn so as to be tangential to the pitch curve, with its centre at the cam centre.
- **Pressure angle:** The pressure angle is the angle between the direction of the follower movement and the normal to the pitch curve.
- **Pitch point:** Pitch point corresponds to the point of maximum pressure angle.
- **Pitch circle:** A circle drawn from the cam center and passes through the pitch point is called Pitch circle
- **Stroke:** The greatest distance or angle through which the follower moves or rotates.

5.3 DISPLACEMENT DIAGRAM FOR UNIFORM VELOCITY, S.H.M. AND UNIFORM ACCELERATION AND DECELERATION.

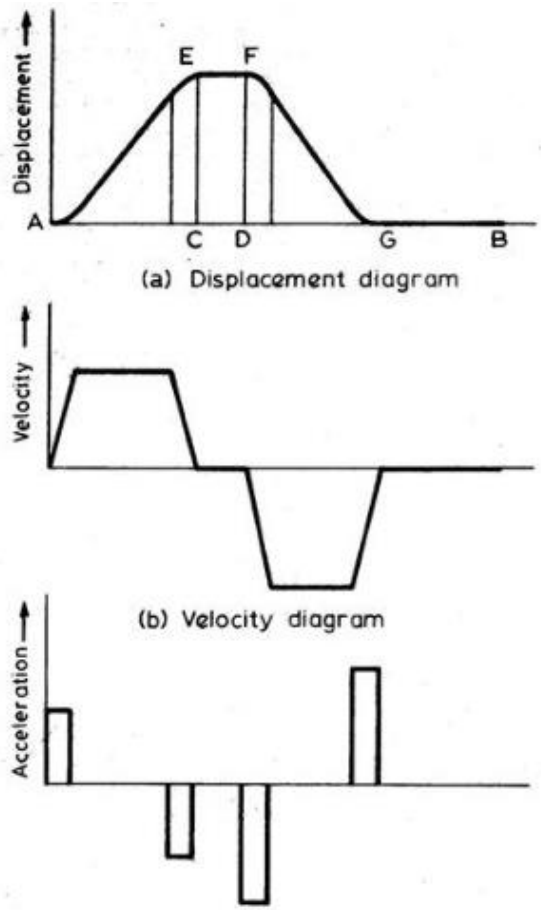**FOLLOWER MOTION WITH UNIFORM VELOCITY**

Figure below shows the displacement, velocity and acceleration patterns of a follower having uniform velocity type of motion. Since the follower moves with constant velocity, during rise and fall, the displacement varies linearly with θ. Also, since the velocity changes from zero to a finite value, within no time, theoretically, the acceleration becomes infinite at the beginning and end of rise and fall.
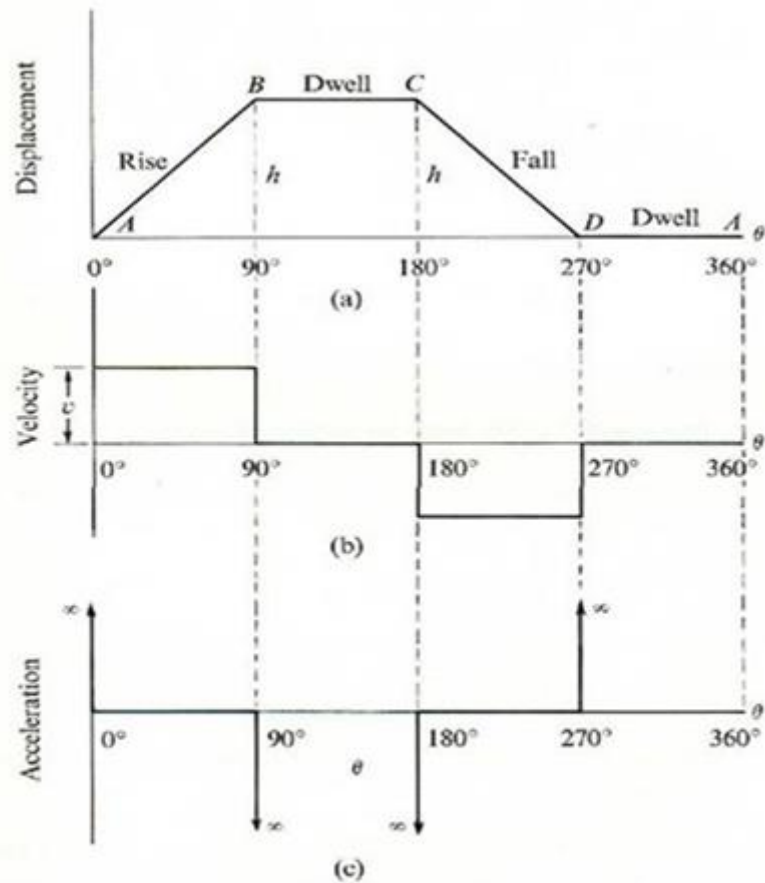
Follower Motion with Uniform Velocity

## FOLLOWER MOTION WITH MODIFIED UNIFORM VELOCITY

It is observed in the displacement diagrams of the follower with uniform velocity that the acceleration of the follower becomes infinite at the beginning and ending of rise and return strokes. In order to prevent this, the displacement diagrams are slightly modified. In the modified form, the velocity of the follower changes uniformly during the beginning and end of each stroke. Accordingly, the displacement of the follower varies parabolically during these periods. With this modification, the acceleration becomes constant during these periods, instead of being infinite as in the uniform velocity type of motion. The displacement, velocity and acceleration patterns are shown in figure below.

Follower Motion with Modified Uniform Velocity

## FOLLOWER MOTION WITH UNIFORM ACCELERATION AND RETARDATION (UARM)

Here, the displacement of the follower varies parabolically with respect to angular displacement of cam. Accordingly, the velocity of the follower varies uniformly with respect to angular displacement of cam. The acceleration/retardation of the follower becomes constant accordingly. The displacement, velocity and acceleration patterns are shown in figure below.
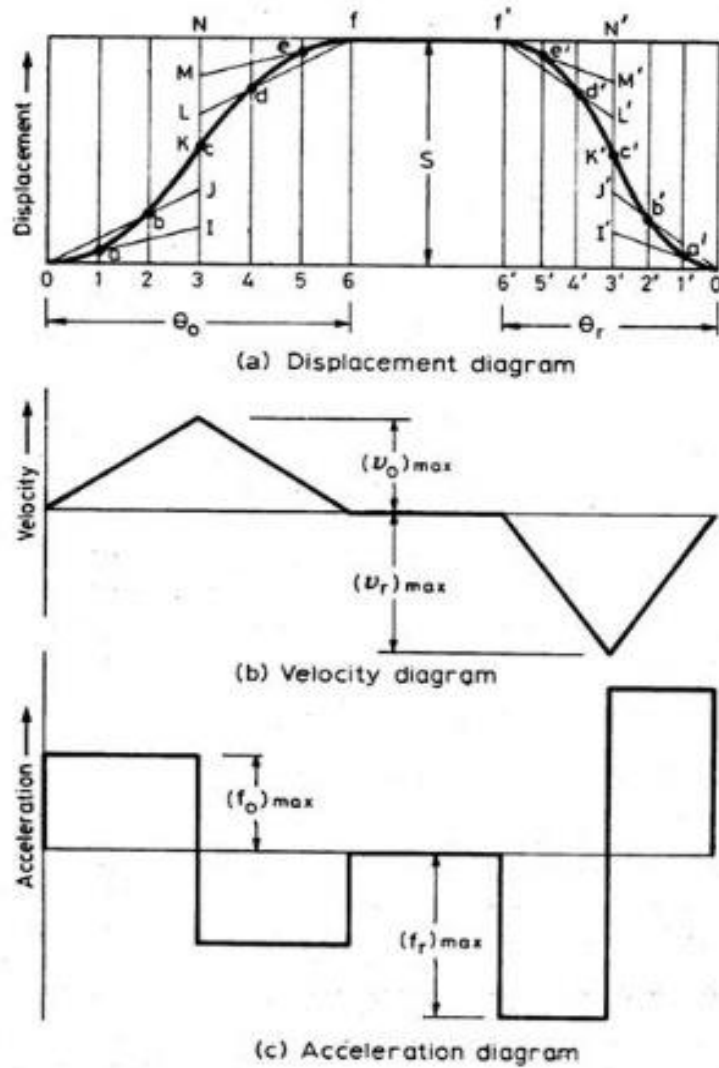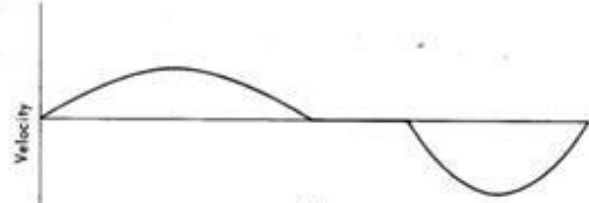
Follower motion with uniform acceleration and retardation
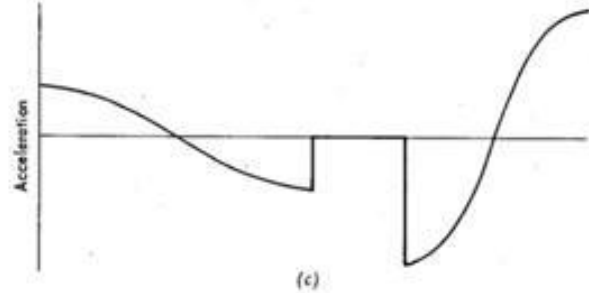
FOLLOWER WITH SIMPLE HARMONIC MOTION (SHM)

In fig.5, the motion executed by point Pl, which is the projection of point P on the vertical diameter is called simple harmonic motion. Here, P moves with uniform angular velocity ω_p, along a circle of radius r (r = s/2).
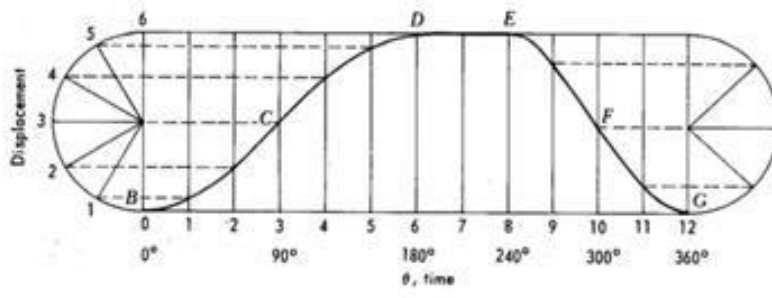
Follower Displacement Diagram with Simple Harmonic Motion

CHAPTER-6

BALANCING

6.1 NEED OF BALANCING, CONCEPT OF STATIC AND DYNAMIC BALANCING

NEED OF BALANCING

The balancing of rotating bodies is important to avoid vibration. In heavy industrial machines such as gas turbines and electric generators, vibration can cause catastrophic failure, as well as noise and discomfort. In the case of a narrow wheel, balancing simply involves moving the center of gravity to the center of rotation. For a system to be in complete balance both force and couple polygons should be closed in order to prevent the effect of centrifugal force. If proper balancing is not done then wear and tear done so to avoid wear & tear and for high production proper balancing is done.

CONCEPT OF STATIC AND DYNAMIC BALANCING

Static Balancing: Static balance occurs when the center of gravity of an object is on the axis of rotation. The object can therefore remain stationary, with the axis horizontal, without the application of any braking force. It has no tendency to rotate due to the force of gravity. This is seen in bike wheels where the reflective plate is placed opposite the valve to distribute the center of mass to the center of the wheel.

Dynamic Balancing: Rotating shaft unbalanced by two identical attached weights, which causes a counterclockwise centrifugal couple Cd that must be resisted by a clockwise couple $F\ell = Cd$ exerted by the bearings. The figure is

drawn from the viewpoint of a frame rotating with the shaft, hence the centrifugal forces.

A rotating system of mass is in dynamic balance when the rotation does not produce any resultant centrifugal force or couple. The system rotates without requiring the application of any external force or couple, other than that required to support its weight. If a system is initially unbalanced, to avoid the stress upon the bearings caused by the centrifugal couple, counterbalancing weights must be added. This is seen when a bicycle wheel gets buckled. The wheel will not rotate itself when stationary due to gravity as it is still statically balanced, but will not rotate smoothly as the centre of mass is to the side of the centre bearing. The spokes on a bike wheel need to be tuned in order to stop this and keep the wheel operating as efficiently as possible

6.2 INTRODUCTION TO BALANCING OF ROTATING MASSES IN THE SAME PLANE AND DIFFERENT PLANES (SIMPLE NUMERICAL)

INTRODUCTION TO BALANCING OF ROTATING MASSES

It is paramount to balance a rotating device both statically and dynamically to eliminate common performance problems such as vibrations, noise, and heat which may cause system failure.

The following cases are important from the subject point of view:

- Balancing of a single rotating mass by a single mass rotating in the same plane.
- Balancing of a single rotating mass by two masses rotating in different planes.
- Balancing of Several Masses Rotating in the Same Plane.
- Balancing of different masses rotating in different planes.

Balancing of a Single Rotating Mass by a Single Mass Rotating in the Same Plane:

Consider a disturbing mass m1 attached to a shaft rotating at ω rad/s.

Let r_1 be the radius of rotation of the mass m_1. We know that the centrifugal force exerted by the mass m_1 on the shaft $F_{C1} = m_1. \omega^2. r_1$ ....(1)
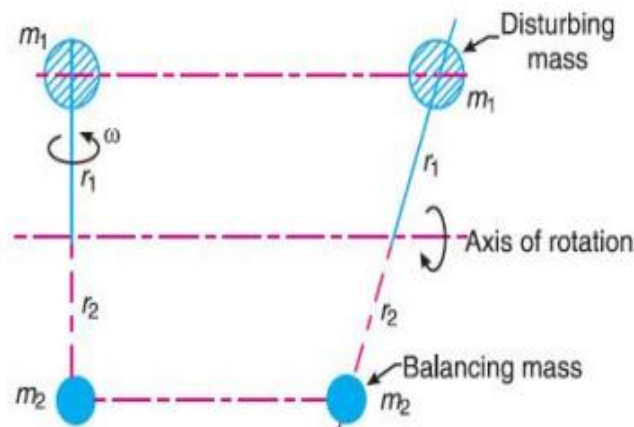
This centrifugal force produces bending moment on the shaft. In order to counteract the effect of this force, a balancing mass (m_2) may be attached in the same plane of rotation as that of disturbing mass (m_1) such that the centrifugal forces due to the two masses are equal and opposite. Let r_2 = Radius of rotation of the balancing mass m_2, centrifugal force due to mass m_2,

$$F_{C2} = m_2. \omega^2. r_2 \quad .... (2)$$

Equating equations (1) and (2),

$m_1. \omega^2. r_1 = m_2. \omega^2. r_2$

or $m_1. r_1 = m_2. r_2$

Balancing of a Single Rotating Mass by Two Masses Rotating in Different Planes: In this case plane of disturbing mass lies in between the planes of the two balancing masses.

Consider a disturbing mass m balanced by two rotating masses m_1 and m_2 as shown in Figure.

Let r, r_1 and r_2 be the radii of rotation of the masses m, m_1 and m_2 respectively.
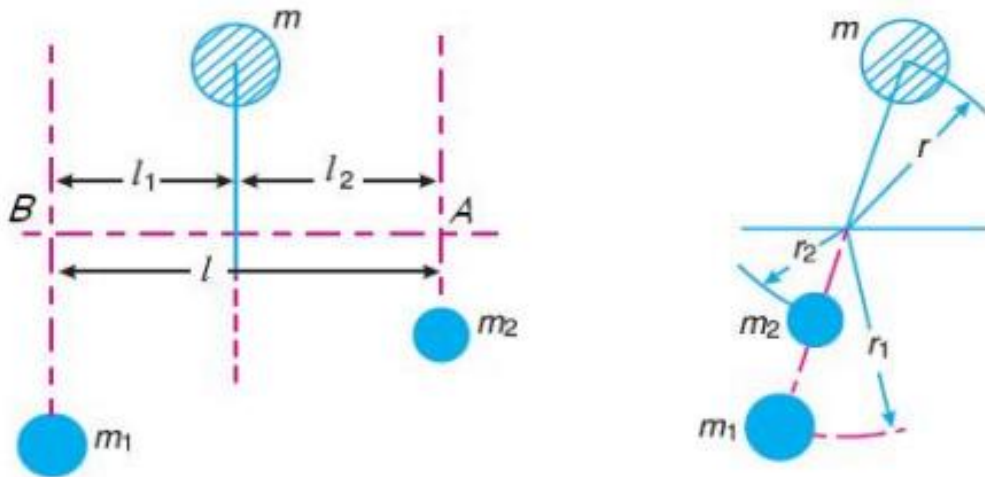
The net force acting on the shaft must be equal to zero

$F_C = F_{C1} + F_{C2}$

$m. \omega^2. r = m_1. \omega^2. r_1 + m_2. \omega^2. r_2$

Or $m. r = m_1. r_1 + m_2. r_2$

Now in order to find the magnitude of balancing force at the bearing B of a shaft, take moments about A. Therefore

$F_{C1} \times 1 = F_C \times l_2$

$m_1. \omega^2. r_1 \times 1 = m. \omega^2. r \times l_2$

$m_1. r_1 \times 1 = m. r \times l_2$

Similarly, in order to find the balancing force at the bearing A of a shaft, take moments about B. Therefore

$F_{C2} \times 1 = F_C \times l_1$

$m_2. \omega^2. r_2 \times 1 = m. \omega^2. r \times l_1$

$m_2. r_2 \times 1 = m. r \times l_1$

Balancing of Several Masses Rotating in the Same Plane:

Let us consider four masses m_1, m_2, m_3 and m_4 rigidly attached to the shaft and revolving about the axis of the shaft. The corresponding radii of rotation are r_1, r_2, r_3 and r_4. Let θ_1, θ_2, θ_3 and θ_4 be the angle of these masses with the horizontal line OX. Let ω be the angular velocity of the shaft in rad/s. The magnitude and

position of the balancing mass may be found out by two methods; (i) Analytical method and (ii) Graphical method as discussed below:

1. Analytical method :

(i) Find out the centrifugal force exerted by each mass on the rotating shaft.

(ii) Resolve each force horizontally and vertically i.e. to find ΣH and ΣV.

As ω^2 is same for each mass.

$\therefore$ $\qquad \Sigma H = m_1 r_1 \cos \theta_1 + m_2 r_2 \cos \theta_2 + m_3 r_3 \cos \theta_3 + m_4 r_4 \cos \theta_4$

Similarly, $\qquad \Sigma V = m_1 r_1 \sin \theta_1 + m_2 r_2 \sin \theta_2 + m_3 r_3 \sin \theta_3 + m_4 r_4 \sin \theta_4$

(iii) Magnitude of resultant centrifugal force

$$F_c = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$$

(iv) If θ' is the angle, which the resultant force makes with the horizontal, then

$$\tan \theta' = \frac{\Sigma V}{\Sigma H}$$

Therefore the angle of balancing mass with the horizontal, $\theta = 180° + \theta'$

(v) The balancing force is equal to the resultant force, but in opposite direction.

(vi) Now, find out the magnitude of balancing mass, such as

$\qquad F_c = mr \qquad\qquad$ [$\because \omega^2$ is same for each mass]

where, $\qquad m$ = balancing mass and r = its radius of rotation.

2. Graphical method :

(i) First, draw the space diagram with the position of several masses as shown in Fig.

(ii) Find out the centrifugal force exerted by each mass of the rotating shaft.

(iii) Now, draw the vector diagram with the obtained centrifugal force such that *ab* represents centrifugal force exerted by mass m_1 in magnitude and direction to some suitable scale. Similarly, draw *bc*, *cd* and *de* representing centrifugal forces of other masses, m_2, m_3 and m_4.

(iv) Now, as per polygon law of forces, the closing side *ae* represents the resultant force in magnitude and direction as shown in Fig.

(v) The balancing force is then equal to the resultant force, but in opposite direction.

(vi) Now, find out the magnitude of balancing mass (*m*), at a given radius of rotation (*r*), such that

$$m\omega^2 r = \text{resultant centrifugal force, } F_c$$

or

$$mr = \text{resultant of } m_1 r_1, \, m_2 r_2, \, m_3 r_3 \text{ and } m_4 r_4$$
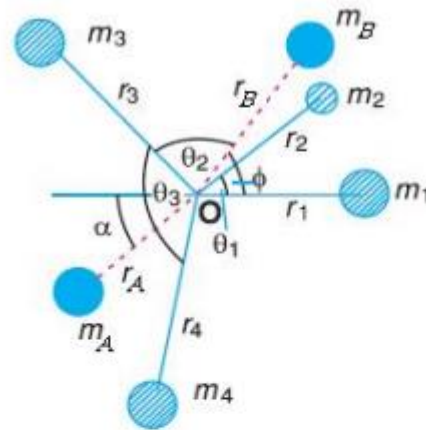
or

$$mr = \text{Resultant vector } ae$$
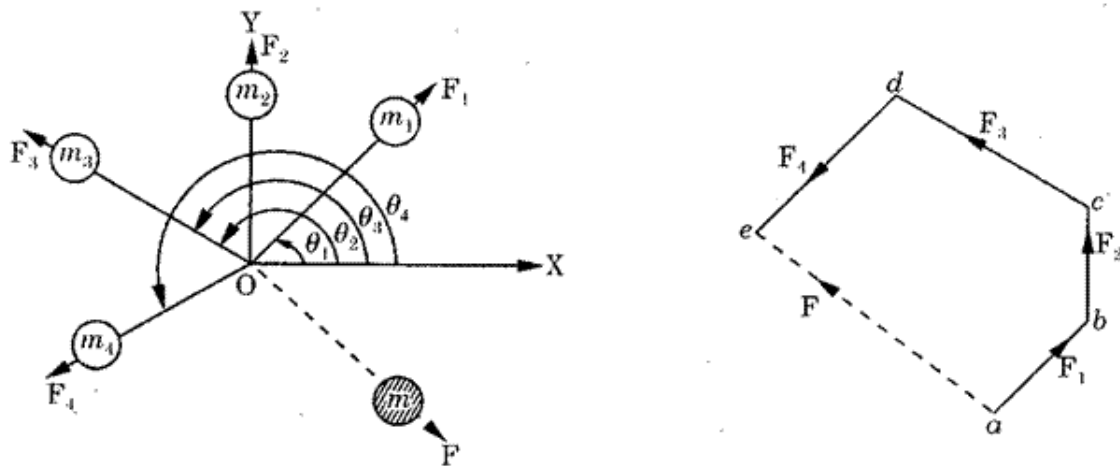
## Balancing of Several Masses Rotating in Different Planes:

Let us consider four masses m_1, m_2, m_3 and m_4 revolving in planes 1, 2, 3 and 4 respectively as shown in Figure (a). The relative angular positions of these masses are shown in Figure (b). The magnitude of the balancing masses m_A and m_B in planes A and B may be obtained as discussed below:
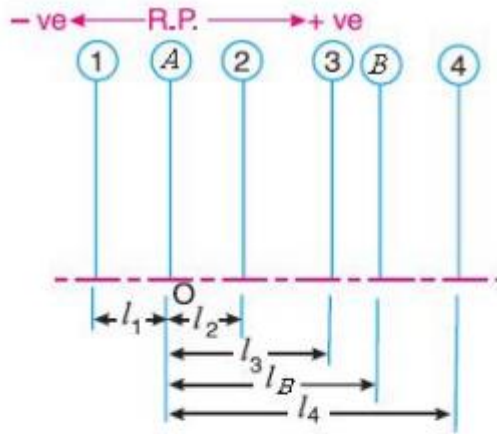
(a) Position of planes of the masses.   (b) Angular position of the masses.
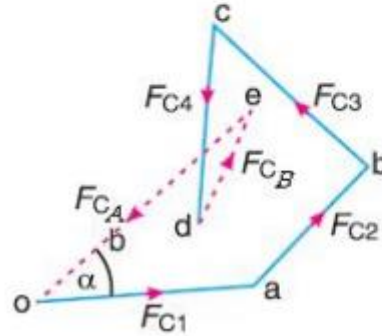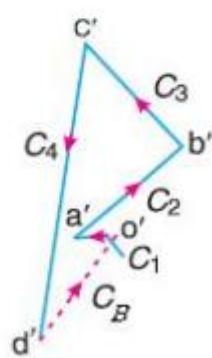
(c) Couple polygon.   (d) Force polygon.

1. Take one of the planes; say A as the reference plane (R.P.).

2. Tabulate the data as shown in Table: 2.

3. The couples about the reference plane must balance, i.e. the resultant couple must be zero.

$$\sum_{i=1}^{n} m_i\, r_i\, l_i\, \cos\theta_i + m_B\, r_B\, l_B \cos\theta_B = 0$$

$$\sum_{i=1}^{n} m_i\, r_i\, l_i\, \sin\theta_i + m_B\, r_B\, l_B \sin\theta_B = 0$$

The forces in the reference plane must balance, i.e. the resultant force must be zero.

$$\sum_{i=1}^{n} m_i\, r_i\, \cos\theta_i + m_A\, r_A\, \cos\theta_A = 0$$

$$\sum_{i=1}^{n} m_i\, r_i\, \sin\theta_i + m_A\, r_A\, \sin\theta_A = 0$$

Example: 1

Four masses m_1, m_2, m_3 and m_4 are 200 kg, 300 kg, 240 kg and 260 kg respectively. The corresponding radii of rotation are 0.2 m, 0.15 m, 0.25 m and 0.3 m respectively and the angles between successive masses are 45°, 75° and 135°. Find the position and magnitude of the balance mass required, if its radius of rotation is 0.2 m.
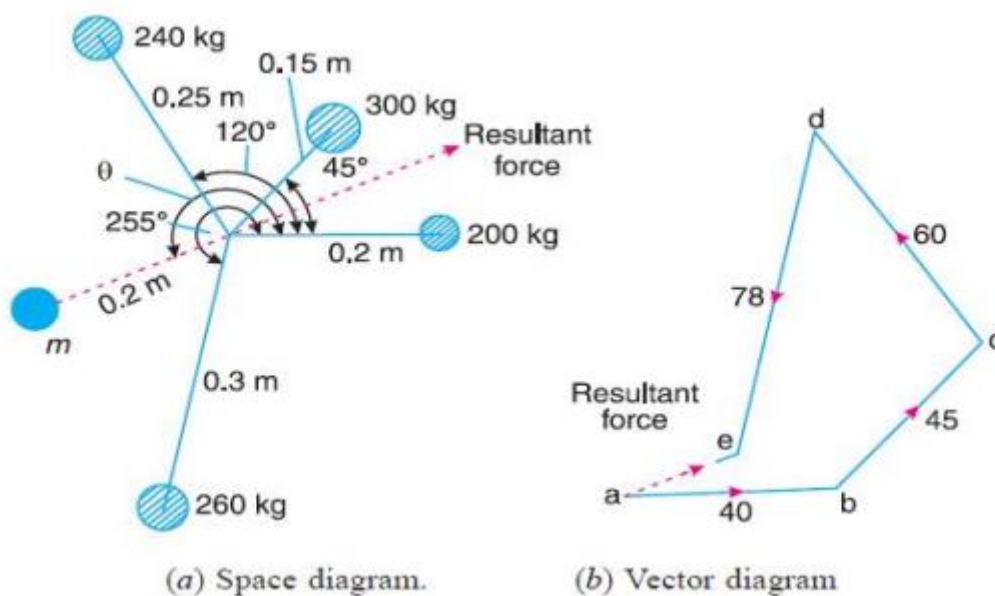
Solution:

Let m = Balancing mass, and
       θ = The angle which the balancing mass makes with m_1.

1. Analytical method

$$\Sigma H = \sum_{i=1}^{n} m_i r_i \cos \theta_i$$

$$\Sigma H = 40 \cos 0 + 45 \cos 45 + 60 \cos 120 + 78 \cos 255 = 21.63 \text{ kg.m}$$

(a) Space diagram.   (b) Vector diagram

No. of masses	Mass (kg)	Radius (m)	Angle(deg.)	Centrifugal force ÷ ω^2 (kg.m)
1	200	0.2	0	40
2	300	0.15	45	45
3	240	0.25	120	60
4	260	0.3	255	78
5	m	0.2	θ	$m.r$

$$\Sigma V = \sum_{i=1}^{n} m_i\, r_i\, \sin\theta_i$$

$$\Sigma V = 40\sin0 + 45\sin45 + 60\sin120 + 78\sin255 = 8.43 \text{ kg.m}$$

$$F_C = \sqrt{(2163.2)^2 + (843.9)^2} = 23.21 \text{ kg.m}$$

$$F_C = m.r, \qquad 23.21 = 0.2 \times m \quad \therefore \qquad m = 116 \text{ kg} \qquad \textbf{Ans.}$$

$$\tan\acute{\theta} = \frac{\Sigma V}{\Sigma H} = \frac{8.43}{21.63} \qquad \therefore \qquad \acute{\theta} = 21.3°$$

Since $\acute{\theta}$ is the angle of the resultant R from the horizontal mass of 200 kg, therefore the angle of the balancing mass from the horizontal mass of 200 kg,

$$\theta = 180° + 21.48° = 201.48° \qquad \textbf{Ans.}$$

Example: 2

A shaft carries four masses A, B, C and D of magnitude 200 kg, 300 kg, 400 kg and 200 kg respectively and revolving at radii 80 mm, 70 mm, 60 mm and 80 mm in planes measured from A at 300 mm, 400 mm and 700 mm. The angles between the cranks measured anticlockwise are A to B 45°, B to C 70° and C to D 120°. The balancing masses are to be placed in planes X and Y. The distance between the planes A and X is 100 mm, between X and Y is 400 mm and between Y and D is 200 mm. If the balancing masses revolve at a radius of 100 mm, find their magnitudes and angular positions.
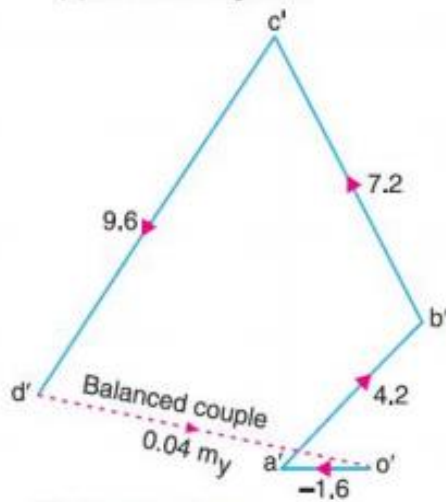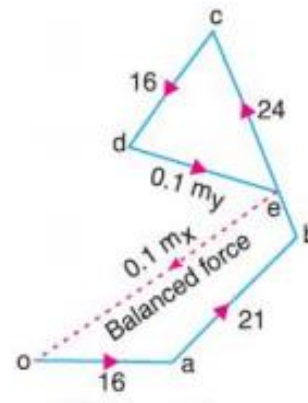
Solution
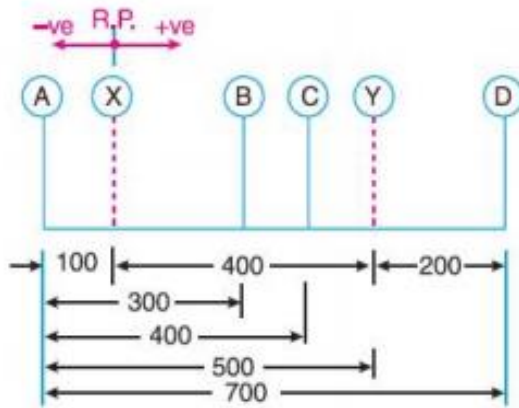
(a) Position of planes.
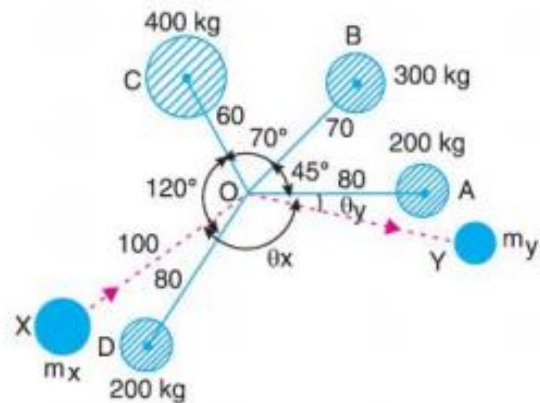
(b) Angular position of masses.

All dimensions in mm.

(c) Couple polygon.

(d) Force polygon.

Plane (1)	Mass (m) kg (2)	Radius (r) m (3)	Cent.force ÷ ω² (m.r) kg-m (4)	Distance from Plane x(l) m (5)	Couple ÷ ω² (m.r.l) kg-m² (6)
A	200	0.08	16	− 0.1	− 1.6
X(R.P.)	m_X	0.1	$0.1\, m_X$	0	0
B	300	0.07	21	0.2	4.2
C	400	0.06	24	0.3	7.2
Y	m_Y	0.1	$0.1\, m_Y$	0.4	$0.04\, m_Y$
D	200	0.08	16	0.6	9.6